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Improved group sampling plans based on time-truncated life tests

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Abstract

In this paper, we consider two new attributes group sampling plans for time truncated life tests. We propose improved single and double group sampling plans based on the total number of failures from the whole groups. The design parameters of the proposed plans are determined using the two-point approach such that the producer's and consumer's risks are satisfied simultaneously at the acceptable reliability level and the lot tolerance reliability level, respectively. The case of the Weibull distribution is described to illustrate the procedure that can be used when the mean life is expressed by a multiple of the specified life. Tables are constructed for various combinations of group size and quality level. The advantage of the proposed plan is shown by comparing with the existing single and two-stage group sampling plans in terms of the average sample number.

Keywords: Acceptance sampling · Average sample number · Consumer's risk · Group sampling · Producer's risk.

Mathematics Subject Classification: Primary 62N05 · Secondary 90B25.

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1. Introduction

An acceptance sampling plan is an inspecting procedure in statistical quality control or reliability tests, which is used to make decisions of accepting or rejecting lots of products to be submitted. This procedure is important for industrial and business purposes of quality management. Acceptance sampling plans have many applications in the field of industries and bio-medical sciences. For their application in food industry one can refer to Bray and Lyon (1973). The main concern in an acceptance sampling plan is to minimize the cost and time required for the quality control or reliability tests for the decision about the acceptance or rejection of the submitted lot of products. The other purpose of acceptance sampling plans is to provide the desired protection to producers and consumers. Producer's and consumer's risks are always attached with the acceptance sampling schemes. A plan which is used to protect both is called a well-designed acceptance sampling plan. In life tests, the cost of testing an item could be very high. The acceptance sampling plan with smaller sample size is called a more economical plan. Single samplings are widely used for these purposes. Double sampling plans are used when we cannot reach the decision on the basis of the first sample. Double sampling plans have advantages over single sampling plans in terms of operating characteristics and the average sample number (ASN).

Attributes single sampling plans based on truncated life tests have been proposed for a variety of life distributions by many authors; see, e.g., Goode and Kao (1961) for the Weibull distribution, Gupta and Groll (1961) for the gamma distribution, Gupta (1962) for normal and log-normal distributions, Kantam et al. (2001) for the log-logistic distribution, Tsai and Wu (2006) for the generalized Rayleigh distribution, and Balakrishnan et al. (2007) for the generalized Birnbaum-Saunders distribution.

Sometimes, testers accommodating multiple items are available in practice because testing time and cost can be saved by testing those items simultaneously. Sudden death testing is frequently adopted by using this type of testers; see Pascual and Meeker (1998) and Jun et al. (2006). For this type of testers, the number of items to be equipped in a tester is given by the specification. If we refer the items in a tester as a group, then we need to determine the number of groups because the group size is already given. This type of acceptance sampling plan is called a group sampling plan. Recently, this type of sampling plans for the truncated life test was proposed by Aslam and Jun (2009a) for the inverse Rayleigh and log-logistic distributions considering only the consumer's risk. Aslam and Jun (2009b) and Aslam et al. (2009) designed group sampling plans for the Weibull and gamma distributions by considering the producer's and consumer's risks at the same time. Aslam et al. (2010) proposed a two-stage group sampling plan for the Weibull distribution, which improved the results given in Aslam and Jun (2009b) in terms of the ASN.

The purpose of this paper is to propose new types of group sampling plans for truncated life tests. Single and a double group sampling plans are constructed based on the total number of failures from all groups under testing. We use the two-point approach when designing the proposed plans. Two cases are considered: (i) one of which is when the acceptable reliability level and the lot tolerance reliability level are expressed by the unreliability, and (ii) the other one is when the quality levels are expressed by the mean ratio to the specified life under the Weibull distribution. The rest of the paper is organized as follows. A single group sampling plan based on the total number of failures is given in Section 2. Specifically, the design parameters indexed by acceptable reliability level (ARL) and lot tolerance reliability level (LTRL) as unreliability are mentioned and parameters for the Weibull distributions are reported in this section. A double group sampling plan based on the total number of failures is proposed in Section 3, where once again the parameters indexed by ARL and LTRL are given and those for the Weibull distributions are obtained. One case study is introduced as a possible application in Section 4. Some concluding remarks are given in Section 5.

2. Group Sampling Plan Based on Total Number of Failures

In this section, a single group sampling plan based on the total number of failures is provided. In the existing group sampling plans, such as those given in Aslam and Jun (2009a,b), a lot under inspection is accepted if the number of failures in each group is smaller than or equal to a specified number. Thus, a lot may be rejected even though the total number of failures is relatively small. Motivated by this, we propose a group sampling plan based on the total number of failures. It is assumed that the capacity of each tester is pre-specified as r items and that its full capacity is used. The algorithm of this plan is the following:

(i) Extract a random sample of size n from a lot;
(ii) Allocate r items to each of g groups (or testers) so that $n = r\,g$;
(iii) Put r items on test before the termination time t_0;
(iv) Accept the lot if the total number of failures from g groups is smaller than or equal to c; and
(v) Truncate the test and reject the lot as soon as the total number of failures from g groups is greater than c before t_0.

The above plan is called single group sampling plan. It is important to note that the proposed plan is a generalization of the ordinary single acceptance sampling plan. This plan becomes the ordinary single sampling plan when $r = 1$. We are interested in finding the design parameters, such as the number of groups g and the action number c, as well as to satisfy the producer's and consumer's risks for given values of the group sizes and true quality levels. The probability of rejecting a good lot is called the producer's risk, which is denoted by α. The probability of accepting a bad lot is called the consumer's risk, which is denoted by β. A lot is accepted if the total number of failures from all groups is smaller than or equal to the specified acceptance number c. Thus, the lot acceptance probability for the proposed plan is given by

$$L(p) = \sum_{i=0}^{c} \binom{rg}{i} p^i (1-p)^{rg-i},$$

where p is the probability that an item in a group fails before the termination time t_0. For a justification of using the binomial distribution in acceptance sampling plans, the reader may refer to Stephens (2001, p. 43).

2.1 Design of single group sampling plan indexed by ARL and LTRL

We adopt the two-point method to determine the design parameters of the plan such that the lot acceptance probability must simultaneously satisfy the specified producer's and consumer's risks; see, e.g., Fertig and Mann (1980). Producers want the probability of accepting should be greater than $1-\alpha$ at the ARL, say p_1. Consumers desire the probability of acceptance should be smaller than β at the LTRL, say p_2. The ARL and the LTRL are in fact unreliabilities at time t_0, which are assumed to be specified. Then, we want to find the design parameters such that the inequalities

$$L(p_1) = \sum_{i=0}^{c} \binom{rg}{i} p_1^i (1-p_1)^{rg-i} \geq 1-\alpha \quad \text{and} \quad L(p_2) = \sum_{i=0}^{c} \binom{rg}{i} p_2^i (1-p_2)^{rg-i} \leq \beta$$

are satisfied. Table 1 shows the design parameters (g and c) for the proposed single group sampling plans indexed by choosing ARLs and LTRLs when the group size is $r = 5$ or $r = 10$. It is assumed that $\alpha = 0.05$ and $\beta = 0.1$.

Table 1. Proposed single group sampling plans indexed by ARL and LTRL.

p_1	p_2			$r = 5$				$r = 10$	
(ARL)	(LTRL)	g	c	Sample size	$L(p_1)$	g	c	Sample size	$L(p_1)$
0.001	0.005	267	3	1335	0.9534	134	3	1340	0.9529
	0.010	107	2	535	0.9829	54	2	540	0.9825
	0.015	52	1	260	0.9716	26	1	260	0.9716
	0.020	39	1	195	0.9834	20	1	200	0.9825
	0.030	26	1	130	0.9923	13	1	130	0.9923
0.005	0.025	54	3	270	0.9522	27	3	270	0.9522
	0.050	21	2	105	0.9839	11	2	110	0.9819
	0.100	8	1	40	0.9828	4	1	40	0.9828
	0.150	5	1	25	0.9931	3	1	30	0.9901
0.010	0.050	27	3	135	0.9526	16	4	160	0.9770
	0.100	11	2	55	0.9822	6	2	60	0.9776
	0.200	4	1	20	0.9831	2	1	20	0.9831
	0.300	3	1	15	0.9904	2	1	20	0.9831
0.050	0.250	5	3	25	0.9659	4	3	30	0.9844
	0.500	2	2	10	0.9885	2	1	10	0.9885
0.100	0.500	3	4	15	0.9873	4	2	20	0.9568

From Table 1, when comparing the case $r = 5$ with the case $r = 10$, it is seen that the acceptance number is determined similarly and that slightly higher sample size is required as the group size increases at the same conditions. Note that the design parameters can be determined independently of the underlying life distribution as long as the ARL and the LTRL are specified. We can see that as the values of LTRL increases for the same value of ARL. The design parameters g and c decrease and the sample size required for the testing purpose decreases too. We cannot find any specific trends in lot acceptance probabilities.

2.2 Design of single group sampling plan for the Weibull distribution

In life testing, it is very often to specify the reliability in terms of the mean life of the lot. Therefore, a life distribution would be applied for this purpose. The Weibull distribution is commonly used in reliability analysis because it includes increasing, decreasing or constant failure rates; for more details about the use of the Weibull distribution in reliability analysis, the reader can refer to Fertig and Mann (1980) and Aslam and Jun (2009b). We consider the Weibull model as a life distribution with known shape parameter m. Normally, the producer records the estimated values of the shape parameter of his(her) product from engineering knowledge. If the shape parameter of the Weibull distribution is unknown, it can be estimated from the historical failure time data. The unreliability can be expressed by the cumulative distribution function (cdf) of the Weibull distribution, which is given by

$$F(t) = 1 - \exp\left(-\left[\frac{t}{\lambda}\right]^m\right), \quad t \geq 0, \tag{1}$$

where λ is an unknown scale parameter and we recall m is a known shape parameter. The mean life of a Weibull distributed item is given as $\mu = (\lambda/m)\,\Gamma\,(1/m)$. Then, the unreliability at time t_0 can be obtained from Equation (1) by

$$p = 1 - \exp\left(- \left[\frac{\Gamma(1/m)}{m} \right]^m \left[\frac{t_0}{\mu} \right]^m \right). \tag{2}$$

It would be convenient to specify the termination time t_0 as a multiple of the specified life μ_0. Specifically, we consider $t_0 = a\mu_0$ for a constant a. Then, the unreliability in Equation (2) reduces to

$$p = 1 - \exp\left(- \left[\frac{a\,\Gamma(1/m)}{m} \right]^m \left[\frac{\mu_0}{\mu} \right]^{-m} \right).$$

If r_1 is the ARL as mean ratio at the producer's risk and r_2 is the LTRL at the consumer's risk, then design parameters should be obtained by satisfying the inequalities

$$L(p_1|\mu/\mu_0 = r_1) \geq 1 - \alpha \quad \text{and} \quad L(p_2|\mu/\mu_0 = r_2) \leq \beta.$$

We consider $r_2 = 1$ because the acceptance of a lot should indicate that the true mean life is greater than the specified mean life at the risk of β.

Tables 2 and 3 were constructed by using Weibull distributions with the shape parameters $m = 2$ and $m = 3$, respectively, for various values of β and mean ratios r_1, two group sizes ($r = 5$ and $r = 10$) and two termination times ($a = 0.5$ and $a = 1.0$). Tables can be constructed for any value of the shape parameter. A computational program that allows us calculate these values is available from authors upon the request. As expected, the number of groups required increases as the consumer's risk becomes smaller and it decreases as the mean ratio becomes larger. When comparing the results for $m = 2$ with those for $m = 3$, the number of groups required increases slightly as the shape parameter becomes larger.

2.3 Comparison with existing group sampling plans

Note that the proposed group sampling plan significantly reduces the number of groups required as compared with the existing group sampling plan proposed by Aslam and Jun (2009b). Figure 1 compares the numbers of groups required by the proposed plan with the existing group sampling plans when $r = 5$, $a = 0.5$ and $\beta = 0.25$. The producer's risk was again assumed as $\alpha = 0.05$. It is observed from Figure 1 that the number of groups required for the proposed plan is much smaller than the existing plan particularly at lower mean ratios of ARL. As the mean ratio increases, the numbers of groups are getting close to both sampling plans. As an example, let us consider the case of $\beta = 0.25$, $\alpha = 0.05$, $m = 2$, $r = 5$, $a = 0.5$ and $r_2 = 2$. The existing plan proposed by Aslam and Jun (2009b) requires $g = 32$ and $c = 2$. Under this plan, a lot is accepted if the number of failures from each of 32 groups is less than or equal to 2. Thus, under this plan, a lot is rejected even when the first group has 3 failures but the rest of 31 groups have no failures. However, the proposed plan requires $g = 7$ and $c = 4$, so that a lot is accepted if the total number of failures is less than or equal to 4, although the number of required groups is reduced significantly.

Table 2. Proposed single group sampling plans for the Weibull distribution ($m = 2$).

		$r = 5$						$r = 10$					
β	μ/μ_0	$a = 0.5$			$a = 1.0$			$a = 0.5$			$a = 1.0$		
	$= r_1$	g	c	$L(p_1)$	g	c	$L(p_1)$	g	c	$L(p_1)$	g	c	$L(p_1)$
0.25	2	7	4	0.9753	3	5	0.9629	4	4	0.9588	2	7	0.9833
	4	3	1	0.9859	1	1	0.9792	2	1	0.9756	1	2	0.9898
	6	↑	↑	0.9970	↑	↑	0.9955	↑	↑	0.9947	↑	1	0.9813
	8	2	0	0.9698	↑	↑	0.9985	1	0	0.9698	↑	↑	0.9937
	10	↑	↑	0.9806	↑	↑	0.9994	↑	↑	0.9806	↑	↑	0.9974
0.10	2	12	6	0.9684	3	5	0.9629	5	5	0.9684	2	7	0.9833
	4	5	1	0.9629	2	2	0.9898	3	2	0.9942	1	2	0.9898
	6	↑	↑	0.9918	↑	1	0.9813	↑	1	0.9884	↑	1	0.9813
	8	3	0	0.9550	↑	↑	0.9813	↑	↑	0.9961	↑	↑	0.9937
	10	↑	↑	0.9710	1	0	0.9615	2	0	0.9615	↑	↑	0.9974
0.05	2	13	6	0.9644	5	8	0.9762	8	7	0.9622	3	9	0.9696
	4	5	1	0.9629	2	2	0.9898	4	2	0.9872	1	2	0.9898
	6	↑	↑	0.9918	↑	1	0.9813	3	1	0.9884	↑	1	0.9813
	8	↑	↑	0.9973	↑	↑	0.9937	↑	↑	0.9961	↑	↑	0.9937
	10	4	0	0.9615	1	0	0.9615	2	0	0.9615	↑	↑	0.9974
0.01	2	19	8	0.9614	6	9	0.9696	11	9	0.9613	3	9	0.9696
	4	9	2	0.9824	3	2	0.9676	5	2	0.9768	2	3	0.9862
	6	7	1	0.9844	2	1	0.9813	4	1	0.9799	1	1	0.9813
	8	↑	↑	0.9948	↑	↑	0.9937	↑	↑	0.9932	↑	↑	0.9937
	10	5	0	0.9521	↑	↑	0.9974	↑	↑	0.9971	↑	↑	0.9974

The cells with upward arrows (↑) indicate that the same values apply as the above cell.

Table 3. Proposed single group sampling plans for the Weibull distribution ($m = 3$).

		$r = 5$						$r = 10$					
β	μ/μ_0	$a = 0.5$			$a = 1.0$			$a = 0.5$			$a = 1.0$		
	$= r_1$	g	c	$L(p_1)$	g	c	$L(p_1)$	g	c	$L(p_1)$	g	c	$L(p_1)$
0.25	2	10	2	0.9820	2	2	0.9530	5	2	0.9820	2	1	0.9530
	4	4	0	0.9726	1	1	0.9988	2	0	0.9726	1	1	0.9948
	6	↑	↑	0.9918	↑	0	0.9837	↑	↑	0.9918	↑	0	0.9676
	8	↑	↑	0.9965	↑	↑	0.9931	↑	↑	0.9965	↑	↑	0.9862
	10	↑	↑	0.9982	↑	↑	0.9964	↑	↑	0.9982	↑	↑	0.9929
0.10	2	13	2	0.9644	2	2	0.9530	7	2	0.9571	2	1	0.9530
	4	6	0	0.9591	2	1	0.9948	3	0	0.9591	1	1	0.9948
	6	↑	↑	0.9877	1	0	0.9837	↑	↑	0.9877	↑	0	0.9676
	8	↑	↑	0.9948	↑	↑	0.9931	↑	↑	0.9948	↑	↑	0.9862
	10	↑	↑	0.9973	↑	↑	0.9964	↑	↑	0.9973	↑	↑	0.9929
0.05	2	18	3	0.9819	2	2	0.9530	9	3	0.9819	2	1	0.9530
	4	7	0	0.9525	2	1	0.9948	6	1	0.9968	1	1	0.9948
	6	↑	↑	0.9857	1	0	0.9837	4	0	0.9837	↑	0	0.9676
	8	↑	↑	0.9939	↑	↑	0.9931	↑	↑	0.9931	↑	↑	0.9862
	10	↑	↑	0.9969	↑	↑	0.9964	↑	↑	0.9964	↑	↑	0.9929
0.01	2	23	3	0.9605	4	4	0.9765	12	3	0.9550	4	2	0.9765
	4	15	1	0.9950	2	1	0.9948	8	1	0.9943	1	1	0.9948
	6	11	0	0.9776	↑	0	0.9676	6	0	0.9756	↑	0	0.9676
	8	↑	↑	0.9905	↑	↑	0.9862	↑	↑	0.9896	↑	↑	0.9862
	10	↑	↑	0.9951	↑	↑	0.9929	↑	↑	0.9947	↑	↑	0.9929

The cells with upward arrows (↑) indicate that the same values apply as the above cell.

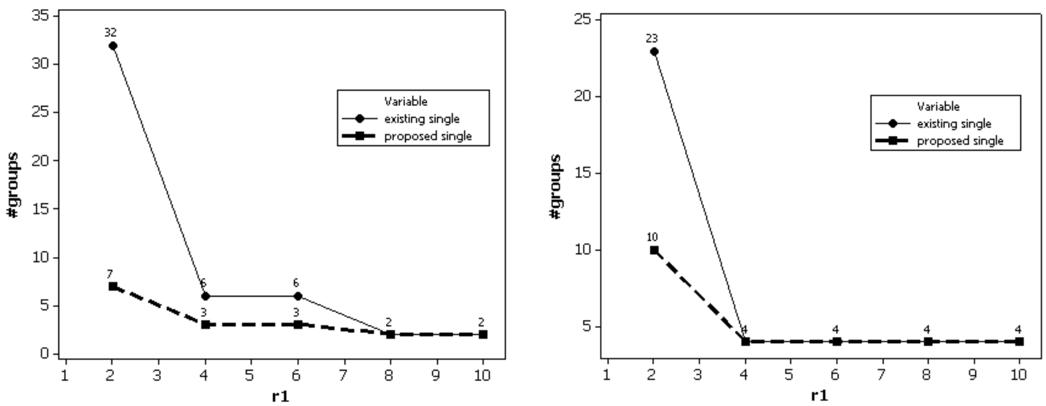

Figure 1. Number of groups in two plans for $m = 2$ (left) and $m = 3$ (right) and $r = 5, a = 0.5$, and $\beta = 0.25$.

3. Improved Double Group Sampling Plan

In this section, a double group sampling plan based on the total number of failures is provided. It is known that a double sampling plan can reduce the sample size required related to a single sampling plan. Thus, we propose a double (or two-stage) group sampling plan for the time truncated life test when using the type of testers with the group size of r. A similar plan has been considered by Aslam et al. (2010), but they are still based on the individual number of failures from each group. However, the proposed plan is based on the total number of failures from all groups. The algorithm of this plan is the following:
First stage:

(i) Draw the first random sample of size n_1 from a lot;
(ii) Allocate r items to each of g_1 groups (or testers) so that $n_1 = rg_1$.
(iii) Put r items on test before the termination time t_0;
(iv) Accept the lot if the total number of failures from g_1 groups is smaller than or equal to c_{1a}; and
(v) Truncate the test and reject the lot as soon as the total number of failures is greater than or equal to $c_{1r}(> c_{1a})$ before t_0. Otherwise, go to the second stage.

Second stage:

(i) Draw the second random sample of size n_2 from a lot;
(ii) Allocate r items to each of g_2 groups so that $n_2 = rg_2$;
(iii) Put r items on test before the termination time t_0;
(iv) Accept the lot if the total number of failures from g_1 and g_2 groups is smaller than or equal to $c_{2a}(\geq c_{1a})$. Otherwise,
(v) Truncate the test and reject the lot.

The proposed double group sampling plan is characterized by five design parameters, namely g_1, g_2, c_{1a}, c_{1r} and c_{2a}. If $c_{1r} = c_{1a} + 1$, then the proposed plan reduces to the single group sampling plan described in Section 2. The number of failures from each group follows a binomial distribution with parameters r and p, where p is the probability that an item in a group fails before the termination time t_0. Thus, the total number of failures from g_1 groups (denoted by X_1) also follows a binomial distribution with parameters n_1 and p. Therefore, the lot acceptance and rejection probabilities at the first stage under the proposed double sampling plan are given by

$$P_a^{(1)} = \mathrm{P}(X_1 \leq c_{1a}) = \sum_{j=0}^{c_{1a}} \binom{n_1}{j} p^j (1-p)^{n_1-j} \quad \text{and} \quad P_r^{(1)} = \sum_{j=c_{1r}}^{n_1} \binom{n_1}{j} p^j (1-p)^{n_1-j}.$$

Now, the lot is accepted from the second stage if the decision has not been made at the first stage and the total number of failures from g_1 and g_2 groups (denoted by X_2) is smaller than or equal to c_{2a}. Hence,

$$\begin{aligned} P_a^{(2)} &= \mathrm{P}(c_{1a}+1 \le X_1 \le c_{1r}-1, X_1+X_2 \le c_{2a}) \\ &= \sum_{x=c_{1a}+1}^{c_{1r}-1} \binom{n_1}{x} p^x (1-p)^{n_1-x} \left[\sum_{i=0}^{c_{2a}-x} \binom{n_2}{i} p^i (1-p)^{n_2-i} \right]. \end{aligned}$$

Therefore, the lot acceptance probability for the proposed double group sampling plan is given by $L(p) = P_a^{(1)} + P_a^{(2)}$.

3.1 Design parameters indexed by ARL and LTRL

When the ARL p_1 and the LTRL p_2 are specified, the design parameters for the proposed double sampling plan can be obtained similarly as in Subsection 2.1. The design parameters satisfying two inequalities may not be unique. Thus, we find the parameters by minimizing the ASN (we prefer the ASN under LTRL to ARL as explained in Aslam et al. (2010)). The ASN under LTRL is given by $\mathrm{ASN}(p_2) = rg_1 + rg_2(1 - P_a^{(1)} - P_r^{(1)})$. Therefore, the optimization problem to be considered is as follows:

$$\begin{aligned} \text{Minimize } \mathrm{ASN}(p_2) &= rg_1 + rg_2(1 - P_a^{(1)} - P_r^{(1)}) \\ \text{Subject to } L(p_1) &\ge 1-\alpha \\ L(p_2) &\le \beta. \end{aligned} \tag{3}$$

The design parameters were determined according to different values of ARL and LTRL and placed in Tables 4 and 5 when the group sizes are $r = 5$ and $r = 10$, respectively. It is assumed that $\alpha = 0.05$ and $\beta = 0.10$. The ASN of the proposed plan as well as the probability of acceptance are reported in this tables. A computational program that allows us calculate these values is available from authors upon the request. From these tables, we observe that the value of c_1 is always determined as zero, whereas the values of c_{1r} and c_{2a} decrease as LTRL increases at a fixed ARL. When comparing the results for $r = 5$ with $r = 10$, the ASN remains quite similar to each other.

Example 3.1 Suppose that a manufacturer wants to adopt the proposed double group sampling plan when making a decision of accepting or rejecting the submitted lots of products. Multi-item testers with group size of 10 are used for the test. He would like to keep the consumer's risk below 10 percent if the unreliability is 0.2, whereas the producer's risk should be less than 5 percent when the unreliability is so low as 0.01. From Table 4, $(c_{1a}, c_{1r}, c_{2a}, g_1, g_2) = (0, 2, 1, 2, 1)$ for the proposed plan, which is implemented following the algorithm:

(i) Take a sample of 20 items from a lot and allocate 10 items to 2 groups;
(ii) Accept the lot if there is no failure in 2 groups or reject the lot if the total number of failures from 2 groups is greater than or equal to 2; otherwise, go to the second stage considering:
(iii) Extract a random sample of size 10 from a lot;
(iv) Allocate the 10 items in 1 group; and
(iv) Accept the lot if the total number of failures is smaller than or equal to 1 from the combined 3 groups.

Table 4. Proposed double group sampling plans indexed by ARL and LTRL

p_1 (ARL)	p_2 (LTRL)	c_{1r}	c_{2a}	g_1	g_2	ASN	$L(p_1)$
		$r = 5$					
0.001	0.005	4	3	139	138	1047.6	0.9508
	0.010	2	2	57	53	328.5	0.9601
	0.015	2	1	34	29	198.7	0.9678
	0.020	2	1	25	23	148.5	0.9809
	0.030	2	1	17	14	98.8	0.9913
0.005	0.025	4	4	36	28	226.0	0.9715
	0.050	2	2	12	9	66.5	0.9587
	0.100	2	1	5	4	29.0	0.9825
	0.150	2	1	4	1	20.7	0.9933
0.010	0.050	4	4	16	16	113.0	0.9743
	0.100	2	2	6	4	32.8	0.9601
	0.200	2	1	3	1	15.7	0.9840
	0.300	2	1	2	1	10.6	0.9913
0.050	0.250	4	3	4	1	21.1	0.9659
	0.500	3	2	2	1	10.3	0.9645
0.100	0.500	4	4	2	1	10.9	0.9807

c_{1a} in all cases is zero.

Table 5. Proposed double group sampling plans indexed by ARL and LTRL

p_1 (ARL)	p_2 (LTRL)	c_{1r}	c_{2a}	g_1	g_2	ASN	$L(p_1)$
		$r = 10$					
0.001	0.005	4	3	70	69	1049.4	0.9502
	0.010	2	2	30	23	334.2	0.9581
	0.015	2	1	17	15	199.7	0.9671
	0.020	2	1	13	10	149.2	0.9814
	0.030	2	1	10	4	105.9	0.9918
0.005	0.025	4	4	17	16	229.2	0.9703
	0.050	3	2	7	4	81.4	0.9826
	0.100	2	1	3	1	31.4	0.9837
	0.150	2	1	2	1	21.4	0.9911
0.010	0.050	4	4	9	8	116.1	0.9688
	0.100	2	2	3	2	32.8	0.9601
	0.200	2	1	2	1	20.6	0.9673
	0.300	2	1	1	1	11.2	0.9870
0.050	0.250	5	4	2	1	24.1	0.9844
	0.500	3	3	1	1	10.5	0.9784
0.100	0.500	4	4	1	1	11.7	0.9535

c_{1a} in all cases is zero.

3.2 Proposed double sampling plan for the Weibull distribution

As in Section 2.2, the design parameters can be determined for the Weibull distribution when the true quality level is expressed by the mean life relative to the specified life. Again, we would like to minimize the ASN at LTRL as in the optimization problem given in Equation (3). Tables 6 and 7 show the design parameters for the Weibull distribution under different combination of group size ($r = 5, 10$) and Weibull shape parameter ($m = 2, 3$). The ASN and the probability of acceptance at the ARL were also given in these tables.

Table 6. Proposed double group sampling plan for Weibull having $m = 2$

β	μ/μ_0	$r=5$						$r=10$					
		c_{1r}	c_{2a}	g_1	g_2	ASN	$L(p_1)$	c_{1r}	c_{2a}	g_1	g_2	ASN	$L(p_1)$
0.25	2	4	4	4	3	27.4	0.9699	4	4	2	2	39.8	0.9553
	4	2	1	2	2	13.0	0.9811	2	1	1	1	13.0	0.9811
	6	↑	↑	↑	↑	13.0	0.9960	↑	↑	↑	↑	13.0	0.9960
	8	1	-	↑	-	10.0	0.9698	↑	↑	↑	↑	13.0	0.9960
0.10	2	6	5	5	5	42.8	0.9687	5	5	3	2	37.1	0.9645
	4	2	1	3	2	16.7	0.9681	2	1	2	1	20.9	0.9533
	6	↑	↑	↑	↑	16.7	0.9930	↑	↑	↑	↑	20.9	0.9598
	8	1	-	↑	-	15.0	0.9550	↑	↑	↑	↑	20.9	0.9965
0.05	2	6	6	7	6	46.6	0.9634	7	6	4	3	52.4	0.9500
	4	2	1	4	2	20.9	0.9533	2	1	2	1	20.9	0.9533
	6	↑	↑	↑	↑	20.9	0.9895	↑	↑	↑	↑	20.9	0.9598
	8	↑	↑	↑	↑	20.9	0.9965	↑	↑	↑	↑	20.9	0.9965
0.01	2	7	8	10	9	58.5	0.9587	7	9	6	4	62.9	0.9641
	4	2	2	5	5	26.0	0.9545	3	2	3	2	31.5	0.9780
	6	↑	1	↑	3	25.6	0.9825	2	1	↑	1	30.2	0.9810
	8	↑	↑	↑	↑	25.6	0.9941	↑	↑	↑	↑	30.2	0.9936

(1) The cells with upward arrows (↑) indicate that the same values apply as the above cell. (2) The cells with hyphens (-) indicate that parameters are irrelevant. (3) c_{1a} in all cases are zeros.

Table 7. Proposed double group sampling plan for Weibull having $m = 3$

β	μ/μ_0	$r=5$						$r=10$					
		c_{1r}	c_{2a}	g_1	g_2	ASN	$L(p_1)$	c_{1r}	c_{2a}	g_1	g_2	ASN	$L(p_1)$
0.25	2	2	2	5	4	30.0	0.9647	2	2	3	1	31.9	0.9554
	4	1	-	4	-	20.0	0.9726	1	-	2	-	20.0	0.9726
	6	↑	-	↑	-	20.0	0.9918	↑	-	↑	-	20.0	0.9918
	8	↑	-	↑	-	20.0	0.9965	↑	-	↑	-	20.0	0.9965
0.10	2	3	2	7	6	46.2	0.9674	3	2	4	3	48.9	0.9599
	4	1	-	6	-	30.0	0.9591	1	-	3	-	30.0	0.9591
	6	↑	-	↑	-	30.0	0.9877	↑	-	↑	-	30.0	0.9877
	8	↑	-	↑	-	30.0	0.9948	↑	-	↑	-	30.0	0.9948
0.05	2	3	3	10	8	57.1	0.9724	3	3	5	4	57.1	0.9724
	4	1	-	7	-	35.0	0.9525	2	1	4	2	42.1	0.9971
	6	↑	-	↑	-	35.0	0.9857	1	-	↑	-	40.0	0.9837
	8	↑	-	↑	-	35.0	0.9938	↑	-	↑	-	40.0	0.9931
0.01	2	4	3	13	11	75.0	0.9566	4	3	7	5	77.0	0.9561
	4	2	1	11	7	56.3	0.9939	2	1	6	2	60.5	0.9947
	6	1	-	↑	-	55.0	0.9939	1	-	↑	-	60.0	0.9756
	8	↑	-	↑	-	55.0	0.9905	↑	-	↑	-	60.0	0.9896

(1) The cells with upward arrows (↑) indicate that the same values apply as the above cell. (2 The cells with hyphens (-) indicate that parameters are irrelevant. (3) c_{1a} in all cases are zeros.

We observed that as mean ratio increases, the values of design parameters tend to decrease. Interestingly, the proposed double group sampling plan reduces to the proposed single group sampling plan when the mean ratio is quite high. Thus, the parameters, such as c_{2a} and g_2, are irrelevant. In these tables, the irrelevant parameters are indicated by hyphens (-). The ASN decreases for same values of a and r when the shape parameter increases.

3.3 Advantage of the Proposed Double Group Sampling Plan

Figure 2 compares the ASN of the proposed double acceptance sampling plans with the proposed single group sampling plan and the existing two-stage group sampling plan discussed in Aslam et al. (2010) for the Weibull distribution. The group size is chosen by $r = 5$ and the termination time is $a = 0.5$. The producer's risk is again 0.05, whereas the consumer's risk is chosen as $\beta = 0.25$. We may construct some other cases as well. From this figure, we can see that proposed double plan provides the smaller ASN as compared with the proposed single sampling plan and the existing two-stage group sampling plan. As an example, when $r_2 = 2$ and $m = 2$, the ASN for the existing two-stage plan is 59.9, while this is 27.4 from the existing plan. Thus, the proposed approach is more economic to save the cost and time of experiment.

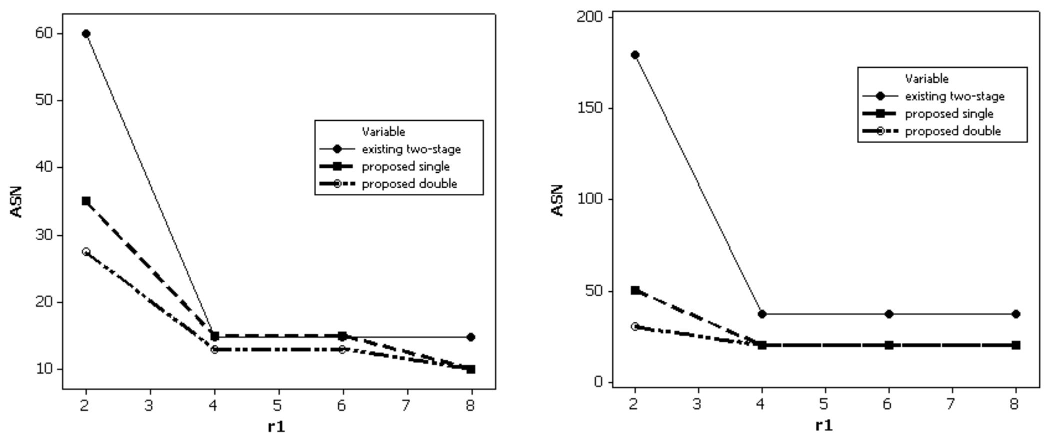

Figure 2. Comparison of three plans in terms of ASN for $m = 2$ (left) and $m = 3$ (right) and $r = 5, a = 0.5$, and $\beta = 0.25$.

4. Application

In this section, one case study is introduced in order to illustrate the results obtained in the paper. Specifically, suppose that a manufacturer of energy saver bulbs would like to design an acceptance sampling plan to decide about the acceptance of the submitted lots. The minimum mean life required for the product is 8,000 hours, i.e., $\mu_0 = 8,000$. Thus, a lot should be accepted if there is sufficient evidence that the true mean life of a product exceeds 8,000 hours. The consumer's risk is chosen as $\beta = 0.1(10\%)$ when the true mean life equals 8,000. The producer's risk is chosen as $\alpha = 0.05(5\%)$ when the true mean life equals 16,000 hours, i.e., $r_1 = 2$. A truncated life test using testers with capacity of 5 products is performed. The test duration is limited by 4,000 hours, i.e., $a = 0.5$.

The lifetime of this product is known to follow the Weibull distribution. In order to estimate its shape parameter, failure data were collected from 10 products of the previous lots as follows: 507, 720, 892, 949, 1031, 1175, 1206, 1428, 1538, 1983. Then, the maximum likelihood estimate of the shape parameter is obtained by $\hat{m} = 2.87$. Thus, let us assume $m = 3$.

Now, let us assume that the manufacturer wants to adopt the proposed single group sampling plan. As $r = 5$, $m = 3, a = 0.5, \beta = 0.1$ and $r_1 = 2$, the design parameters can be found from Table 3 and they are chosen as $(g, c) = (13, 2)$. This sampling plan is implemented as follows:

(i) Extract a random sample of 65 energy saver bulbs from a lot;
(ii) Allocate the 5 bulbs in each one of 13 testers so that $n = 5 \times 13 = 65$;
(iii) Put 65 bulbs on test before the termination time $t_0 = 4,000$ hours;
(iv) Accept the lot if the total number of failures from 13 groups is smaller than or equal to $c = 2$ during the test termination time $t_0 = 4,000$ hours; and
(v) Truncate the test and reject the lot as soon as the total number of failures from 13 groups reaches 3 before the test ends.

Table 3 also shows that the actual acceptance probability is 0.9644 under this condition, which means that there is still 3.56% risk of rejecting the lot having mean life 16,000 hours.

Now consider the case that the manufacturer may choose the proposed double group sampling plan. Then, the design parameters are found from Table 7 as $(c_{1a}, c_{1r}, c_{2a}, g_1, g_2) = (0, 3, 2, 7, 6)$. This plan is implemented as follows:

(i) Extract a random sample of 35 energy saver bulbs from a lot;
(ii) Allocate the 5 bulbs in each one of 7 testers so that $n = 5 \times 7 = 35$;
(iii) Put 35 bulbs on test before the termination time $t_0 = 4,000$ hours;
(iv) Accept the lot if the total number of failures from 7 groups is $c = 0$ during the test termination time $t_0 = 4,000$ hours;
(v) Truncate the test and reject the lot if the total number of failures from 7 groups reaches 3 before the test ends; otherwise (if the total number of failures during the test is 1 or 2), go to the second stage considering:
(vi) Extract a random sample of 30 bulbs from the lot;
(vi) Allocate the 30 bulbs in 6 groups; and
(iv) Accept the lot if the total number of failures is smaller than or equal to 2 from the combined 13 groups, but reject the lot, otherwise.

Thus, when the manufacturer adopts the proposed double group sampling plan, the ASN is 46.2, which is smaller than the sample size (65) for the proposed single group sampling plan.

5. Concluding Remarks

In this paper, we have proposed two new group sampling plans on the basis of the total number of failures from all groups. We have determined the design parameters, such as the number of groups and the acceptance number, for two cases, which are: (i) for the specified ARL and the LTRL and (ii) for the Weibull distribution. We can conclude that the proposed double group sampling plan performs better than the existing two-stage group sampling plan and the proposed single group sampling plan in this study in terms of the ASN. We have observed that the proposed double group sampling plan reduces to the proposed single group sampling plan when the ARL is specified high enough.

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