Chilean Journal of Statistics
Vol. 1, No. 2, September 2010, 3–15

Special Issue "In Memory of Pilar Loreto Iglesias Zuazola"
Research Paper

Portfolio selection: An application to the Chilean stock market

Reinaldo B. Arellano-Valle[1,*], Heleno Bolfarine[2], Pilar L. Iglesias[1] and Paola Viviani[3]

[1]Departamento de Estadística, Pontificia Universidad Católica de Chile, Santiago, Chile
[2]Departamento de Estatística, Universidade de São Paulo, São Paulo, Brazil
[3]Departamento de Salud Pública, Pontificia Universidad Católica de Chile, Santiago, Chile

Remembering Pili for her joy in working with statistics

(Received: 12 March 2010 · Accepted in final form: 25 August 2010)

Abstract

In this paper, we consider the problem of estimating the systematic risk of stocks market by using a modeling formulation based on scale normal mixtures comparative calibration models. In this work, we emphasize the Student-t comparative calibration model, which is approached by considering the degrees of freedom parameter unknown. Inference is approached by using the EM algorithm and MCMC methodology. The results are applied to the stock returns of two Chilean companies.

Keywords: Bayesian inference · Capital asset pricing model · Maximum likelihood estimation · Scale normal mixture · Structural comparative calibration model · Student-t distribution.

Mathematics Subject Classification: Primary 90A09 · Secondary 62H12.

1. Introduction

The main object of this paper is to estimate the parameters of the capital asset pricing model (CAPM) by using the comparative calibration model and historical data from the Chilean stock market, with emphasis on the problem of estimating the systematic risk. The CAPM specifies that the expected return of an asset (or share) is equal to the free risk rate plus the price of the risk; see, e.g., Sharpe (1964), Lintner (1965), Fama (1965), Mossin (1966) and Elton and Gruber (1995). Specifically, let R denotes the random variable expressing the asset return so that the CAPM model establishes that

$$\mathrm{E}(R) = r_f + \beta(\mathrm{E}(R_m) - r_f), \tag{1}$$

*Corresponding author. Reinaldo B. Arellano-Valle. Departamento de Estadística, Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Avenida Vicuña Mackenna 4860, Macul, Santiago, Chile. Email: reivalle@mat.puc.cl

ISSN: 0718-7912 (print)/ISSN: 0718-7920 (online)

where R denotes the share return, r_f is the risk free return rate, β is the systematic risk of the asset under study and R_m is the market return generally given as an index.

Much interest on the field of financial economics is focused on the efficient estimation of the parameters of the return generating process. The estimator of β in the above model is an important measure of risk for financial analysis and also for risk and portfolio managers. This parameter is very useful for calculating the capital of equities cost, a key quantity for evaluation methods. An extension of the model given in (1), which is typically used for estimating the coefficient β corresponding to an asset, is

$$\mathrm{E}(R) - r_f = \alpha + \beta(\mathrm{E}(R_m) - r_f), \tag{2}$$

where α is the asset return independent of the market. The estimation of the parameter β of the model given in (2) has been approached by considering the regression model

$$r_j - r_{fj} = \alpha + \beta(r_{mj} - r_{fj}) + \varepsilon_j$$

or

$$Y_j = \alpha + \beta x_j + \varepsilon_j,$$

where r_j denotes the assets' return at the j-th period, r_{mj} is the market return at the j-th period and r_j corresponds to the risk free return at the j-th period, so that

$$Y_j = r_j - r_{fj}$$

and

$$x_j = r_{mj} - r_{fj},$$

represent the return of an asset in excess of risk free rate and the excess return of the market portfolio of assets at the j-th period, respectively, for $j = 1, \ldots, n$.

The estimation approach that has been considered in the financial literature is based on the least squares theory under the assumption that the random errors $\varepsilon_1, \ldots, \varepsilon_n$ are independent and identically distributed according to the $\mathrm{N}(0, \sigma^2)$, i.e., the normal distribution with mean zero and variance σ^2. It is also known that least squares is highly sensitive to extreme observations which are typically present in the Chilean stock market. Duarte and Mendes (1997) presented robust methods for estimating β in Latin American emerging markets. Cademartori et al. (2003) approached the problem from a classical point of view of independent Student-t random errors. Using the EM-algorithm, McLachlan and Krishnan (1997) obtained maximum likelihood (ML) estimators of β for several Chilean companies. Bayesian solutions to the problem have been considered by Harvey and Zhou (1990) and Shanken (1987), under the normality assumption, with applications to the US market.

Along these lines, robust methods are developed by replacing the normal distribution by symmetrical distributions with heavier tails than the normal, which allows reducing the influence of outliers on estimators.

The use of heavy tailed distributions such as the Student-t distribution in usual regression models has been investigated in Lange et al. (1989) and Geweke (1993); see also Polson and Tew (1999).

The present paper is different from previous ones in several aspects due to: (i) the market return is considered as a non-observable random variable, which is expressed as the sum of the asset's return with an error term; (ii) the information on the daily returns of several companies are combined in a comparative calibration model; (iii) heavy tailed distributions are considered for the error terms; and (iv) a comparative study is reported comparing classical and Bayesian solutions. Finally, the main results are applied to the Chilean stock market, where extreme observations are often encountered. Additionally, the Bayesian approach allows the consideration of informative priors for the model parameters.

Comparative calibration models have been considered to compare measuring instruments, which are used for obtaining measurements from the same unknown quantity. Such models are special cases of multivariate measurement error models; see, e.g., Fuller (1987). Some applications in education, industry, medicine and psychology have been considered in Grubbs (1948, 1973), Barnett (1969), Carter (1981), Dunn (1989), Bolfarine and Galea-Rojas (1995, 1996), among others.

The comparative calibration model is defined as follows. Let x_j denotes the true value of the unknown quantity corresponding to the j-th individual (sample unit), for $j = 1, \ldots, n$, and Y_{ij}, the measurement that follows by using the i-th instrument, for $i = 1, \ldots, n$. With a linear relationship between Y_{ij} and x_j (Barnett, 1969; Kimura, 1992), we consider the model

$$Y_{ij} = \alpha_i + \beta_i x_j + \varepsilon_{ij}, \quad i = 1, \ldots, p, \, j = 1, \ldots, n.$$

The parameters α_i and β_i represent the additive and multiplicative bias corresponding to i-th instrument, for $i = 1, \ldots, p$. Barnett (1969) considered the existence of a reference instrument (namely instrument 0), which makes unbiased measurements, that is, $\alpha_0 = 0$ and $\beta_0 = 1$. The reference instrument typically corresponds to the most precise instrument. Thus, the model we consider can be represented as

$$Y_{0j} = x_j + \varepsilon_{0j} \tag{3}$$

and

$$Y_{ij} = \alpha_i + \beta_i x_j + \varepsilon_{ij}, \tag{4}$$

for $i = 1, \ldots, p$ and $j = 1, \ldots, n$, where x_j, as before, represents the true (unobserved) market return at the j-th period, corrected for the free rate of return, Y_{ij} the return of the asset corresponding to company i at time j also corrected for the free rate of return. Since the x_j is considered as a random variable, the model that we consider is a structural one; see, e.g., Bolfarine and Galea-Rojas (1995). As pointed out by a referee, the above model may be adequate under special market conditions such as market equilibrium. A model taking into consideration trends in returns is studied in Aguilar and West (2000).

The paper is organized as follows. In Section 2, we present ML estimation of the parameters for the comparative calibration model given in (3) and (4), under the assumption of Student-t random errors (normal model as a limiting case), with fixed degrees of freedom. In Section 3, we consider a Bayesian approach for normal mixtures comparative calibration model including the Student-t model with known and unknown degrees of freedom as special cases. In Section 4, we discuss applications to the Chilean stock market, including also a comparative study between classical and Bayesian approaches. Finally, we provide some conclusions in the final part.

2. Maximum Likelihood Estimation

As discussed in the introduction part, the general comparative calibration model is defined by (3) and (4). The normal structural model, where $x_j \overset{\text{i.i.d.}}{\sim} \mathrm{N}(\mu_x, \sigma_x^2)$ and $\varepsilon_{ij} \overset{\text{i.i.d.}}{\sim} \mathrm{N}(0, \sigma_i^2)$, with x_j independent of ε_{ij}, for $j = 1, \ldots, n$ and $i = 0, \ldots, p$, has been considered in, e.g., Barnett (1969) and Bolfarine and Galea-Rojas (1996). In this case, it is possible to obtain explicit expressions for the ML estimator of $\theta = (\alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_p, \mu_x, \sigma_x, \sigma_0, \ldots, \sigma_p)^\top$ when $p = 2$. For $p > 2$, we need numerical algorithms to find the solutions. For $p = 1$, some additional conditions on θ are required in order to avoid identifiability problems; see Barnett (1969) and Fuller (1987). For a more compact representation of this model, let $Y_j = (Y_{0j}, Y_{1j}, \ldots, Y_{pj})^\top$, for $j = 1, \ldots, n$. Thus, the normal structural model is defined by

$$Y_j \overset{\text{ind.}}{\sim} \mathrm{N}_{p+1}(\mu, \Sigma), \quad j = 1, \ldots, n, \tag{5}$$

where $\mu = \mu(\theta)$ and $\Sigma = \Sigma(\theta)$ are given by

$$\mu = \begin{pmatrix} \mu_x \\ \alpha_1 + \beta_1 \mu_x \\ \vdots \\ \alpha_p + \beta_p \mu_x \end{pmatrix} \tag{6}$$

and

$$\Sigma = \begin{pmatrix} \beta_p \sigma_x^2 + \sigma_0^2 & \beta_1 \sigma_x^2 & \cdots & \beta_p \sigma_x^2 \\ \beta_1 \sigma_x^2 & \beta_1^2 \sigma_x^2 + \sigma_1^2 & \cdots & \beta_1 \beta_p \sigma_x^2 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_p \sigma_x^2 & \beta_1 \beta_p \sigma_x^2 & \cdots & \beta_p^2 \sigma_x^2 + \sigma_p^2 \end{pmatrix}, \tag{7}$$

respectively. Under the normal structural model given in (5) with $p \geq 2$, the ML estimator of μ and Σ are the sample mean vector

$$\bar{Y} = \frac{1}{n} \sum_{j=1}^{n} Y_j$$

and sample covariance matrix

$$S = \frac{1}{n} \sum_{j=1}^{n} (Y_j - \bar{Y})(Y_j - \bar{Y})^\top,$$

respectively. Hence, the ML estimator of θ is the solution (contained in the parametric space of θ) to the equations:

$$\mu(\hat{\theta}) = \bar{Y} \quad \text{and} \quad \Sigma(\hat{\theta}) = S. \tag{8}$$

We consider in this section a more general situation than model given in (5) by assuming that

$$Y_j \overset{\text{i.i.d.}}{\sim} t_{p+1}(\mu, \Sigma; \nu), \quad j = 1, \ldots, n, \tag{9}$$

where $\mu = \mu(\theta)$ and $\Sigma = \Sigma(\theta)$ are as in Equations (6) and (7) and $t_{p+1}(\mu, \Sigma; \nu)$ denotes the $(p+1)$-dimensional Student-t distribution with location vector μ, dispersion matrix Σ and ν degrees of freedom. The model given in (9) is equivalent to considering, in (3) and (4), that the random vectors

$$(x_j, \varepsilon_{0j}, \varepsilon_{1j}, \dots, \varepsilon_{pj})^\top \overset{\text{i.i.d.}}{\sim} t_{p+1}(\xi, \Omega, \nu), \quad j = 1, \dots, n,$$

where $\xi = (\mu_x, 0, 0, \dots, 0)^\top$ and $\Omega = \mathrm{diag}\{\sigma_x^2, \sigma_0^2, \sigma_1^2, \dots, \sigma_p^2\}$. As is well known, the normal structural model given in (5) can be obtained as a limiting case of the Student-t structural model given in (9) by letting $\nu \to \infty$.

From model given in (9) and considering the degrees of freedom known, the likelihood function $\mathrm{L}(\theta) = f(Y_1, \dots, Y_n \mid \theta)$ is such that

$$\mathrm{L}(\theta) \propto |\Sigma|^{-n/2} \prod_{j=1}^{n} \left\{1 + \frac{1}{\nu}(Y_j - \mu)^\top \Sigma^{-1}(Y_j - \mu)\right\}^{-(\nu+p+1)/2}, \tag{10}$$

where θ is defined above, with $\mu = \mu(\theta)$ and $\Sigma = \Sigma(\theta)$ defined in Equations (6) and (7), respectively. Expression given in (10) is analytically complicated to deal with. Hence, to solve the maximization problem associated with model given in (9), we make use of the Monte Carlo EM algorithm (Tanner, 1996) using the fact that the model given in (9) can be alternatively specified in two steps:

(i) $Y_j | w_j \sim \mathrm{N}_{p+1}(\mu, w_j\Sigma)$ and

(ii) $w_j \overset{\text{i.i.d.}}{\sim} \mathrm{IGa}\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$, for $j = 1, \dots, n$, where $\mathrm{IGa}(a, b)$ denotes the inverted gamma distribution with shape parameter a and scale parameter b.

The model defined in step (i) corresponds to the heteroscedastic normal model, with variances differing within individuals, i.e., conditionally on the missing vector $w = (w_1, \dots, w_n)^\top$. The log-likelihood function corresponding to θ is such that

$$\begin{aligned} \log\big(f(Y_1, \dots, Y_n \mid \theta, w)\big) &= \sum_{j=1}^{n} \log\big(f(Y_j|\theta, w_j)\big) \\ &\propto -\frac{n}{2}\log(|\Sigma|) - \frac{1}{2}\sum_{j=1}^{n}\log(w_j) - \frac{1}{2}\sum_{j=1}^{n}\frac{1}{w_j}(Y_j - \mu)^\top \Sigma^{-1}(Y_j - \mu), \end{aligned} \tag{11}$$

where, as before, $\mu = \mu(\theta)$ and $\Sigma = \Sigma(\theta)$ are defined in Equations (6) and (7). The Monte Carlo EM algorithm (Tanner, 1996) is applied to the situation consisting of computing initial values θ^0 from expressions given in (11) using the initial values $w^0 = (w_1^0, \dots, w_n^0)^\top$ for the missing vector $w = (w_1, \dots, w_n)^\top$. From this stage, we proceed as follows. At step i,

(a) Generate $w_1^i, \dots, w_n^i$ from the distribution of $w|Y_1, \dots, Y_n, \theta^i$ and

(b) Find $\theta^{i+1} = \underset{\theta}{\mathrm{argmax}} \left\{\frac{1}{n}\sum_{t=1}^{n} \log\left(f(Y_j|\theta, w_j^i)\right)\right\}$,

iterating until convergence, which is checked by looking at the difference $|\theta_j^{i+1} - \theta_j^i|$.

To implement the E-step of the algorithm, it is required to obtain the conditional distribution of w_j given $Y_1, \ldots, Y_n, \theta$. Straightforward computations lead to

$$w_j | Y_j, \theta \overset{\text{ind.}}{\sim} \text{IGa}\left(\frac{\nu + p + 1}{2}, \frac{(Y_j - \mu)\,\Sigma^{-1}\,(Y_j - \mu) + \nu}{2}\right), \quad j = 1, \ldots, n. \tag{12}$$

To compute standard deviations for the ML estimators, the inverse of minus the Hessian matrix that follows from the likelihood function Equation (10), evaluated at the ML estimates, will be used. This evaluation can also be done numerically.

Remark 2.1 The above results can be applied without additional difficulties to other representable models such as mixtures of normal distributions. It suffices to specify a distribution G for w_i as considered in Arellano-Valle et al. (2000) and Branco et al. (2000). Furthermore, in the case of the Student-t model, the approach can be made more robust by considering the degrees of freedom ν as an unknown parameter. An approximation to the ML estimator can be computed by varying ν over a grid of values and considering, as an estimator of ν, the value in the grid that maximizes the likelihood function.

3. Bayesian Analysis

In this section, we consider a Bayesian solution for the inference problem associated with model given in (3) and (4) under the hypothesis of mixtures of normally distributed errors, which has as a special case the normal and Student-t distributions. For the Student-t model, we consider the two situations where the degrees of freedom are considered known and unknown. More specifically, we consider the model given in (3) and (4) under the following assumptions:

$$x_j | w_j \overset{\text{ind.}}{\sim} \text{N}(\mu_x, w_j \sigma_x^2), \quad \varepsilon_{ij} | w_j \overset{\text{ind.}}{\sim} \text{N}(0, w_j \sigma_i^2), \quad w_j \overset{\text{i.i.d.}}{\sim} G, \tag{13}$$

for $i = 0, \ldots, p$ and $j = 1, \ldots, n$, all of them conditionally independent given w_j, where G represents the mixing distribution. For the Student-t structural model, G corresponds to the gamma distribution with parameters $\nu/2$ and $\nu/2$, denoted by $\text{Ga}(\nu/2, \nu/2)$.

To perform the Bayesian analysis, we implement the MCMC type approach since a full Bayesian approach is difficult to carry out on such models. One special MCMC type approach which requires only the specification of the conditional posterior distributions for each parameter is the Gibbs sampler. In situations where those distributions are simple to sample from, as in the present case, the approach is easily implemented. In other situations, the more complex Metropolis-Hastings approach needs to be considered; see Gamerman and Lopes (2006). Thus, to implement the Gibbs sampler approach, we need to obtain the conditional posterior distributions. Considering that α_i, β_i and σ_i^2 are a priori independent, the following prior distributions are considered:

$$\alpha_i \overset{\text{ind.}}{\sim} \text{N}(a_{0i}, v_{1i}), \quad \beta_i \overset{\text{ind.}}{\sim} \text{N}(b_{0i}, v_{2i}) \quad \text{and} \quad \sigma_i^2 \overset{\text{ind.}}{\sim} \text{IGa}\left(\frac{d_i}{2}, \frac{d_i}{2}\right), \quad i = 0, \ldots, p,$$

where $\alpha_0 = 0$ and $\beta_0 = 1$. Also, if model given in (13) we consider μ_x and σ_x^2 known, then by letting $x = (x_1, \ldots, x_n)^\top$ and $y = (Y_1, \ldots, Y_n)^\top$, we obtain the following conditional distributions:

$$\alpha_i|\alpha_{(i)}, \beta, \sigma^2, x, w, y \sim \mathrm{N}\left(\frac{\sum_{j=1}^{n}\frac{(Y_{ij}-\beta_i x_j)}{w_j} + \frac{a_{0i}\sigma_i^2}{v_{1i}}}{\sum_{j=1}^{n}\frac{1}{w_j} + \frac{\sigma_i^2}{v_{1i}}}, \frac{\sigma_i^2}{\sum_{j=1}^{n}\frac{1}{w_j} + \frac{\sigma_i^2}{v_{1i}}}\right),$$

$$\beta_i|\alpha, \beta_{(i)}, \sigma^2, x, w, y \sim \mathrm{N}\left(\frac{\sum_{j=1}^{n}\frac{(Y_{ij}-\alpha_i)x_j}{w_j} + \frac{b_{0i}\sigma_i^2}{v_{2i}}}{\sum_{j=1}^{n}\frac{x_j^2}{w_j} + \frac{\sigma_j^2}{v_{2i}}}, \frac{\sigma_j^2}{\sum_{j=1}^{n}\frac{x_j^2}{w_j} + \frac{\sigma_j^2}{v_{2i}}}\right),$$

$$\sigma_i^2|\alpha, \beta, \sigma_{(i)}^2, x, w, y \sim \mathrm{IGa}\left(\frac{n+d_i}{2}, \frac{\sum_{j=1}^{n}\frac{(Y_{ij}-(\alpha_i-\beta_i x_j))^2}{w_j} + d_i}{2}\right)$$

and

$$x_j|\alpha, \beta, \sigma^2, x_{(j)}, w, y \sim \mathrm{N}\left(\frac{\sum_{i=1}^{p}\frac{\sigma_x^2}{\sigma_i^2}(Y_{ij}-\alpha_i)\beta_i + \mu_x}{\sum_{i=1}^{p}\frac{\sigma_x^2}{\sigma_i^2}\beta_i^2 + 1}, \frac{w_j\sigma_x^2}{\sum_{i=1}^{p}\frac{\sigma_x^2}{\sigma_i^2}\beta_i + 1}\right),$$

for $i = 0, \ldots, p$ and $j = 1, \ldots, n$, where $\alpha = (\alpha_1, \ldots, \alpha_p)^\top$, $\beta = (\beta_1, \ldots, \beta_p)^\top$, and for any vector, say $u = (u_1, \ldots, u_m)^\top$, with $u_{(i)} = (u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_m)^\top$.

Thus, if we consider $w_j = 1$ for all $j = 1, \ldots, n$, then the normal model follows. On the other hand, if

$$w_j \overset{\text{i.i.d.}}{\sim} \mathrm{IGa}\left(\frac{\nu}{2}, \frac{\nu}{2}\right), \quad j = 1, \ldots, n,$$

then the Student-t model with ν degrees of freedom follows. In this case, the conditional posterior distribution of w_j is

$$w_j|\alpha, \beta, \sigma^2, x, w_{(j)}, y \sim \mathrm{IGa}\left(\frac{p+\nu}{2}, \sum_{i=1}^{p}\frac{(Y_{ij}-\alpha_i-\beta_i x_j)^2}{\sigma_j^2}\right), \quad j = 1, \ldots, n.$$

In the case where ν is unknown with $\nu \sim \mathrm{Exp}(a)$, the conditional posterior is given by

$$\nu|w, y \propto \exp\left\{\sum_{j=1}^{n}\left(\frac{\nu}{w_j} + \left(\frac{\nu}{2}+1\right)\log\left(\frac{\nu}{w_j}\right) - a\nu\right)\right\}.$$

The algorithm starts with initial values for $\alpha_{(i)}$ and x (all zeros, say), and β, σ^2 and w (all ones, say) and cycles, generating samples from the above conditional distributions, until convergence. This could be verified using the approach developed, e.g., by Gelman and Rubin (1992), which requires running several parallel chains. It is also worth noticing that the Bayesian approach does not involve matrix inversion and hence it can deal more adequately with a greater number of assets.

4. Application to the Chilean Stock Market Dataset

In this section, we apply the theory and methods developed earlier to the series of monthly stock returns from two Chilean companies, for the January' 83 - December' 92 period. The first one is COPEC, the Chilean oil company, of big impact in the Chilean stock market. The second is Concha y Toro, a wine manufacturer, with a relatively smaller weight in the financial market. We also have the corresponding set of values of IPSA, the Chilean version of the Dow-Jones index. In this case, $p = 2$ and

$$\begin{aligned} Y_{0j} &= \text{IPSA return in month } j, \\ Y_{1j} &= \text{Concha y Toro return in month } j, \\ Y_{2j} &= \text{COPEC return in month } j, \end{aligned}$$

for $j = 1, \ldots, 120$.

4.1 Maximum likelihood estimation

Table 1 presents the ML estimate of the parameters of the Student-t structural likelihood function given in Equation (10) for some given values of ν. This table suggests that a Student-t structural model with $\nu = 5$ degrees of freedom is the most adequate model to fit the data. This is close to the value ν that maximizes the likelihood function when a simple Student-t regression model for each company is considered (see Table 2).

Table 1. ML estimates for the Student-t structural model, for some given values of ν.

Parameters	$\nu = 2$	$\nu = 4$	$\nu = 5$	$\nu = 6$	$\nu = 10$	$\nu = 50$
α_1	-0.01077	-0.01049	-0.01024	-0.01012	-0.01010	-0.00838
α_2	-0.01769	-0.01845	-0.01829	-0.01826	-0.01908	-0.01952
β_1	0.47466	0.53600	0.56551	0.57899	0.64147	0.82198
β_2	1.20649	1.26012	1.28813	1.29442	1.33970	1.37656
μ_x	0.01819	0.02100	0.02148	0.02202	0.02340	0.05820
σ_x	0.00378	0.00427	0.00437	0.00452	0.00473	0.00555
σ_0	0.00101	0.00124	0.00132	0.00137	0.00154	0.00170
σ_1	0.00375	0.00533	0.00588	0.00638	0.00780	0.01280
σ_2	0.00220	0.00270	0.00287	0.00301	0.00330	0.00424
Likelihood	348	352	353	351	346	329

Table 2 shows ML estimates of β for different degrees of freedom when a simple Student-t regression model is considered for each company. For Concha y Toro the value of ν producing the highest likelihood is $\nu = 1$; while for COPEC, the highest likelihood values is obtained for $\nu = 3$ degrees of freedom. It can also be noted that the estimate of β presents the greatest change for Concha y Toro company, as ν goes from 1 to 200. This effect seems somewhat less pronounced when the Student-t structural model is considered; see Table 1. The estimates of β still increases, but at a less pronounced rate. Summarizing the above, there is strong indication that Concha y Toro presents heavy tails. Similar findings were also reported in Cademartori et al. (2003).

Table 2. ML estimates with standard errors in parenthesis for simple Student-t regression models.

	Concha y Toro		COPEC	
ν	β (standard error)	L(θ)	β (standard error)	L(θ)
1	0.17802 (0.0310)	112.1241	0.88161 (0.0395)	123.0125
2	0.26736 (0.0413)	111.5709	0.92612 (0.0467)	130.4801
3	0.31447 (0.0470)	107.7606	0.95346 (0.0502)	131.3884
4	0.35156 (0.0509)	104.3123	0.97194 (0.0523)	131.1957
5	0.38146 (0.0537)	101.4412	0.9853 (0.0537)	130.7718
10	0.47103 (0.0617)	92.47642	1.01958 (0.0569)	128.9223
50	0.63549 (0.0758)	78.77003	1.05704 (0.0604)	126.0199
100	0.67422 (0.0788)	76.41305	1.06241 (0.0609)	125.5357
200	0.69611 (0.0805)	75.18724	1.06515 (0.0612)	125.2819

4.2 A Bayesian analysis

The Bayes estimates were obtained using the WinBUGS software and the following prior specifications:

$$\alpha_i \sim \mathrm{N}(0, 1/16), \quad \beta_i \sim \mathrm{N}(1, 1/16), \quad i = 1, 2 \quad \text{and} \quad \sigma_i^2 \sim \mathrm{IGa}(0.01, 0.01), \quad i = 0, 1, 2,$$

all they assumed to be independent. The choice of the priors for the α and β parameters was guided to assessments made from market analysis; see Polson and Tew (1999). However, the prior distributions considered for the dispersion parameters are essentially non-informative.

The results for the normal structural model are presented in Tables 3 and 4 for different lags using Gibbs Sampling. For this model, we consider

$$x_j \overset{\text{i.i.d.}}{\sim} \mathrm{N}\left(0, \frac{1}{16}\right), \quad j = 1, \ldots, n.$$

To avoid correlation problems in the generated chains, different lag values were considered. For some chains, the lag values were taken to be 15, for others 20, and so on. However, the values of the posterior means do not seem to experience significant changes. We also point out the fact that risk estimates are smaller than those obtained with the ML approach in the normal comparative calibration model, which is the special case of the Student-t model with large degrees of freedom.

Comparing the estimates above, particularly for the systematic risk β, with estimators for each company computed under the simple normal regression models given in Table 4, one can notice that they are quite close.

The results for the Student-t structural model for different degrees of freedom are presented in Table 5. When a simple Student-t regression model for each company was considered, we obtain similar results as presented in Table 6.

Notice that, in this case and under both models, the systematic risk estimation for Concha y Toro experiments important changes compared to the corresponding estimation for COPEC. This may be caused by the fact that, for COPEC, linearity may be less pronounced under a Student-t model.

The results when we consider a prior distribution $\nu \sim \text{Exp}(0.01)$ are presented in Table 8 for the Student-t structural model, and in Table 9 when a separate simple Student-t regression model for each company is considered. Differences between separate and joint situations may be caused by the more concentration of the later.

Notice that, for the Student-t structural, the Bayesian and classical (maximum likelihood) estimators for the degrees of freedom parameter ν are close, which is expected since non-informative priors were used. The same does not occur with the simple Student-t regression models, particularly for COPEC. In fact, while the classical estimate of ν obtained by fitting a simple Student-t regression model for COPEC (see Table 2) indicates a strong evidence against the normality of this company, the posterior mean of ν obtained under the same simple regression model for COPEC indicates that this company is not necessarily non-normal. However, it is important to recall that the classical estimators for ν considered here are not exactly the maximum likelihood ones.

Table 3. Posterior means and standard deviations for the normal structural model (lag = 1).

Parameters	Posterior mean	Standard deviation
α_1	-0.0010	0.0123
α_2	-0.0099	0.0083
β_1	0.7854	0.1195
β_2	1.0840	0.0881
σ_0	0.0510	0.0075
σ_1	0.1281	0.0088
σ_2	0.0706	0.0089

Table 4. Posterior means and standard deviations for the normal structural model (lag = 15).

Parameters	Posterior mean	Standard deviation
α_1	-0.0019	0.0125
α_2	-0.0109	0.0084
β_1	0.7920	0.1182
β_2	1.0870	0.0865
σ_0	0.0512	0.0076
σ_1	0.1281	0.0088
σ_2	0.0706	0.0086

Table 5. Posterior means for the simple normal regression models.

Parameters	Concha y Toro	COPEC
α	-0.0027	-0.0112
β	0.7863	1.0600
σ	0.5063	0.0875

Table 6. Posterior means and standard deviations for the Student-t structural model (lag = 15).

Parameters	Posterior mean $\nu = 2$	$\nu = 10$	$\nu = 50$
α_1	-0.0090	-0.0073	-0.0037
	(0.0065)	(0.0093)	(0.0115)
α_2	-0.0122	-0.0113	-0.0106
	(0.0073)	(0.0079)	(0.0084)
β_1	0.3667	0.5673	0.7226
	(0.0787)	(0.1020)	(0.1142)
β_2	0.9828	1.0700	1.0870
	(0.0801)	(0.0876)	(0.0852)
σ_0	0.0382	0.0485	0.0505
	(0.0052)	(0.0069)	(0.0074)
σ_1	0.0549	0.0909	0.1173
	(0.0060)	(0.0081)	(0.0086)
σ_2	0.0428	0.0596	0.0677
	(0.0055)	(0.0078)	(0.0085)

Table 7. Posterior means for the simple Student-t regression model.

	Concha y Toro			COPEC		
ν	α	β	σ	α	β	σ
2	-0.0098	0.3541	0.0575	-0.0134	0.9388	0.0549
10	-0.0087	0.5659	0.0943	-0.0119	1.0200	0.0754
50	-0.0045	0.7160	0.1215	-0.0109	1.0500	0.0847
200	-0.0032	0.7639	0.1299	-0.0111	1.0560	0.0867

Table 8. Posterior means and standard deviations for the Student-t structural model (lag = 20).

Parameters	Posterior mean	Standard deviation
α_1	-0.0081	0.0081
α_2	-0.0113	0.0078
β_1	0.4824	0.0977
β_2	1.0450	0.0837
σ_0	0.0446	0.0065
σ_1	0.0753	0.0089
σ_2	0.0531	0.0074
ν	5.0780	1.2470

Table 9. Posterior means for the simple Student-t regression models (lag = 20).

Parameters	Concha y Toro	COPEC
α	-0.0102	-0.0124
β	0.3480	1.0040
σ	0.0561	0.0713
ν	1.9150	12.4300

5. Conclusions

The paper considered Bayesian and classical approaches for estimating the systematic risk using simple regression and structural comparative calibration models under Student-t errors. The main idea was to consider market returns as latent variables, incorporating information from different companies using a comparative calibration model. To control for external effects that may affect the market, specially in the case of small companies, we considered heavy tailed distributions for the error terms, with emphasis on the Student-t distribution. Inference for the proposed models was based on classical (maximum likelihood) and Bayesian approaches. Both approaches can be computationally implemented by using simple and accessible software such as S-Plus and WinBUGS. The results were applied to the Chilean stock market where the main objective was to make inference on the parameter β (the slope of the regression model) for each company and also make inference on the degrees of freedom ν of the Student-t model with the objective of selecting the best model. With the classical approach, the degrees of freedom considered was $\nu = 5$, which was obtained by choosing the value of ν providing the highest value for the Student-t structural likelihood function given in Equation (10). A similar results was obtained from the Bayes estimator of ν (with squared error loss) for the Student-t structural model. However, when separate simple Student-t regression models were considered for each company, the Bayes estimator obtained for ν suggested that the simple normal model regression model (ν large) could be appropriate for COPEC company, but not for Concha y Toro company. Finally, the Bayesian approach provided a posterior distribution for β (systematic risk) that is useful in making investment decisions using a utility function associated with the problem.

Acknowledgements

The work of this paper is partially supported by grant FONDECYT 1085241-Chile and project PROSUL. The authors acknowledge the undergraduate student Carlos Bustos L. for computational support. The authors also thank the editors and a referee for their helpful comments and suggestions.

References

Aguilar, O., West, M., 2000. Bayesian factor models and portfolio allocation. Journal Business and Economic Studies, 18, 338-357.

Arellano-Valle, R.B., Galea-Rojas, M., Iglesias, P., 2000. Bayesian sensitivity analysis in elliptical linear regression models. Journal of Statistical Planning and Inference, 86, 175-199.

Barnett, V.D., 1969. Simultaneous pairwise linear structural relationships. Biometrics, 25, 129-142.

Bolfarine, H., Galea-Rojas, M., 1995. Structural comparative calibration using the EM algorithm. Journal of Applied Statistics, 22, 277-292.

Bolfarine, H., Galea-Rojas, M., 1996. One structural comparative calibration under a t-model. Computational Statistics, 11, 63-85.

Branco, M., Bolfarine, H., Iglesias, P., Arellano-Valle, R.B., 2000. Bayesian analysis of the calibration problem under elliptical distributions. Journal of Statistical Planning and Inference, 90, 69-85.

Cademartori, D., Romo, C., Campos, R., Galea, M., 2003. Robust estimation of systematic risk using the t-distribution in the Chilean stock markets. Applies Economic Letters, 10, 447-453.

Carter, R., 1981. Restricted maximum likelihood estimation of bias and reliability in the comparison of several measuring methods. Biometrics, 37, 733-741.

Duarte, A.M., Mendes, B., 1997. Robust estimation of systematic risk in emerging stock markets. Emerging Markets Quarterly, 1, 85-95.

Dunn, G., 1989. Design and Analysis of Reliability Studies: The Statistical Evaluation of Measurement Errors. Arnold, London.

Elton, E.J., Gruber, M.J., 1995. Modern Portfolio Theory and Investment Analysis. 5th Edition. Wiley, New York.

Fama, E., 1965. The behavior of stock market prices. The Journal of Business, 38, 34-105.

Fuller, W.A., 1987. Measurement Error Models. Wiley, New York.

Gamerman, D., Lopes, H.F., 2006. Markov Chain Monte Carlo: Stochastic Simulation for Bayesian Inference. Second edition. Chapman and Hall/CRC, Boca Raton, FL.

Gelman, A., Rubin, D., 1992. Inference from iterative simulation using multiple sequences, Statistical Science, 7, 457-511.

Geweke, J., 1993. Bayesian treatment of the independent Student-t linear model. Journal of Applied Econometrics, 8S, 19-40.

Grubbs, F.E., 1948. On estimating precision of measuring instruments and product variability. Journal of the American Statistical Association, 43, 243-264.

Grubbs, F.E., 1973. Errors of measurement, precision, accuracy and statistical comparison of measuring instruments. Technometrics, 15, 53-66.

Harvey, C.R., Zhou, G., 1990. Bayesian inference in asset pricing tests. The Journal of Financial Economics, 26, 221-254.

Kimura, D., 1992. Functional comparative calibration using an EM algorithm. Biometrics, 48, 1263-1271.

Lange, K.L., Little, R.J.A., Taylor, J.M.G., 1989. Robust statistical modeling using the t-distribution. Journal of the American Statistical Association, 84, 881-896.

Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, The Review of Economics and Statistics, 41, 13-37.

McLachlan, G., Krishnan, T., 1997. The EM Algorithm and Extensions. Wiley, New York.

Mossin, J., 1966. Equilibrium in capital asset market, Econometrica, 35, 768-783.

Polson, N., Tew, B., 1999. Bayesian portfolio selections: an analysis of the S&P 500 index 1970-1996. Journal of Business & Economic Statistics, 18, 164-173.

Sharpe, W., 1964. Capital asset prices: a theory of markets equilibrium under conditions of risk. Journal of Finance, 19, 425-442.

Shanken, J., 1987. A Bayesian approach to testing portfolio efficiency. Journal of Financial Economics, 19, 195-215.

Tanner, M., 1996. Tools for Statistical Inference. Springer, New York.

Zhou, G., 1993. Asset-pricing tests under alternative distribution. Journal of Finance, 48, 1927-1942.