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Research Paper

Stereographic Logistic Model - Application to Noisy Scrub Birds Data

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Abstract

This paper introduces Stereographic Logistic Model based on inverse Stereographic Projection or Bilinear (Mobious) Transformation. Considering the data set of orientation of 50 noisy scrub birds along the bank of a creck bed, it is shown that the said model is a good fit using Watson's U^2 and Kuiper's tests. The derivation of the characteristic function for Stereographic Logistic Model and its trigonometric moments are presented. Relative performance of Stereographic Logistic, Wrapped Logistic and Stereographic Double Exponential models for the live data of 50 noisy scrub birds is studied. Also graphs of pdf of this new Model for various combinations of the parameters are drawn. Here it is established that the new model is uni/bimodal depending on the concentration parameter, $\sigma < 0.5$ or $\sigma > 0.5$.

Keywords: Characteristic function · Circular models · Concentration parameter · Goodness-of-fit · Inverse sterographic projection.

Mathematics Subject Classification: 60E05 · 62H11.

1. Introduction

Dattatreya Rao et al. (2007) constructed new circular models by applying wrapping by reducing a linear variable to its modulo 2π and using trigonometric moments. Dattatreya Rao and his school consisting of Ramabhadra Sarma, Girija, Phani, Radhika, Devaraaj and Srihari, have introduced several new models and a few new methodologies of constructing new Circular models. As a result, some of the new models came into existence viz. Wrapped Logistic, Wrapped Lognormal, Wrapped Weibull and Wrapped Extreme value models (Dattatreya Rao et al., 2007), (Ramabhadra Sarma et al., 2011), Wrapped Half logistic and Wrapped Binormal models (Ramabhadra Sarma et al., 2009), Wrapped Gamma Distribution (Dattatreya Rao et al., 2013b). Some of stereographic versions of linear models are Stereographic Lognormal (Girija et al., 2013a), Stereographic Extreme value (Phani et al., 2012a), Stereographic Double Weibull (Phani et al., 2015), Stereographic Reflected Gamma (Phani et al., 2012b), Stereographic Arc tan Exponential-Type models (Phani et al., 2014a), Stereographic Double Exponential (Girija et al., 2014a),

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Stereographic Semicircular models (Phani, 2013), Stereographic Circular Normal Moment Distribution (Phani et al., 2014b). A good number of circular models by offsetting bivariate linear distributions are constructed viz. Offset Cauchy model (Girija et al., 2013b), Offset Pearson Type-II model (Radhika et al., 2013a), Offset l-arc models (Girija et al., 2014f), Offset t distribution (Radhika, 2014). Rising Sun Circular models are Rising Sun Cardioid model (Girija, 2010), Rising Sun Wrapped Lognormal and Rising Sun Wrapped Exponential Models (Dattatreya Rao et al., 2013a), Rising Sun von Mises and Rising Sun Wrapped Cauchy models (Radhika et al., 2013b). Differential Equation for Cardioid model (Dattatreya Rao et al., 2011) is also derived. Construction of Discrete Circular models are initiated viz., Wrapped Binomial model (Girija et al., 2014c) and Wrapped Poisson Distribution (Girija et al., 2014d).

Taking this as a cue, here an attempt is made to derive a new circular model by applying the inverse Stereographic projection on the Logistic model. The density, distribution and characteristic functions of the proposed new circular model are derived. Relative performance of the new circular models fitting to 50 noisy scrub birds Data is studied.

2. Methodology of Inverse Stereographic Projection

Inverse Stereographic Projection is defined by a one to one mapping given by $T(\theta) = x = u + \nu\tan(\theta/2)$, where $x \in (-\infty, \infty)$, $\theta \in [-\pi, \pi)$, $u \in \mathrm{R}$ and $\nu > 0$. Then $T^{-1}(x) = \theta = 2\tan^{-1}\{(x-u)/\nu\}$ by Minh and Farnum (2003) is a random point on the unit circle.

Suppose x is randomly chosen on the interval $(-\infty, \infty)$. Let $F(x)$ and $f(x)$ denote the Cumulative distribution and probability density functions of the random variable X respectively. Also $G(\theta)$ and $g(\theta)$ denote the Cumulative distribution and probability density functions of this random point θ on the unit circle respectively. Then $G(\theta)$ and $g(\theta)$ can be written in terms of $F(\theta)$ and $f(x)$ using the following Theorem 2.1 as stated below.

Theorem 2.1 For $\nu > 0$,

i) $G(\theta) = F\big(u + \nu\tan(\theta/2)\big) = F(x(\theta))$,

ii) $g(\theta) = \nu\Big(\frac{1+\tan^2(\theta/2)}{2}\Big) f\big(u + \nu\tan(\theta/2)\big) = \nu\Big(\frac{1+((x(\theta)-u)/\nu)^2}{2}\Big) f(\theta(x))$.

3. Stereographic Logistic Distribution

A random variable X on the real line is said to have Logistic Distribution with location parameter γ and scale parameter $\lambda > 0$, if the probability density function and cumulative distribution function of X for $x, \gamma \in \mathrm{R}$ and $\lambda > 0$ are given by

$$f(x) = \frac{1}{\lambda}\left[1 + \exp\left(\frac{-(x-\gamma)}{\lambda}\right)\right]^{-2} \exp\left(\frac{-(x-\gamma)}{\lambda}\right) = \frac{1}{4\lambda}\operatorname{sech}^2\left(\frac{x-\gamma}{2\lambda}\right),$$

$$F(x) = \left[1 + \exp\left(\frac{-(x-\gamma)}{\lambda}\right)\right]^{-1} = \frac{1}{2}\left[1 + \tanh\left(\frac{1}{2}\left(\frac{x-\gamma}{\lambda}\right)\right)\right],$$

respectively.

Then by applying Inverse Stereographic Projection defined by a one to one mapping $x = u + \nu\tan(\theta/2)$, u, $\nu > 0 \in \mathrm{R}$ and $-\pi \le \theta < \pi$, which leads to a Circular Stereographic Logistic Distribution on unit circle.

Definition 3.1 A random variable X_S on unit circle is said to have Stereographic Logistic

Distribution with location parameter μ and scale parameter $\sigma > 0$ denoted by SLG(μ, σ), if the probability density and cumulative distribution functions are given by

$$g(\theta) = \frac{1}{2\sigma} \sec^2(\theta/2) \left[1 + \exp\left(-\left(\frac{\tan(\theta/2) - \mu}{\sigma}\right)\right)\right]^{-2} \exp\left(-\left(\frac{\tan(\theta/2) - \mu}{\sigma}\right)\right),$$

where $\sigma = \lambda/\nu > 0$, $\mu = \gamma/\nu$ and $-\pi \leq \theta < \pi$

$$G(\theta) = \left[1 + \exp\left(-\left(\frac{\tan(\theta/2) - \mu}{\sigma}\right)\right)\right]^{-1},$$

where $\sigma > 0$, $-\pi \leq \theta < \pi$ respectively.

Hence the proposed new model SGL(μ, σ) is a circular model named by us as "Stereographic Logistic Model".

It can be seen that the Stereographic Logistic Model is Symmetric about $\mu = 0$.

Result 3.2 Stereographic logistic distribution is Symmetric about $\mu = 0$ and is unimodal if $\sigma < 0.5$ and bimodal if $\sigma > 0.5$.

Proof The probability density function of Stereographic Logistic Distribution is

$$g(\theta) = \frac{1}{2\sigma} \sec^2\left(\frac{\theta}{2}\right) \left[1 + \exp\left(-\left(\frac{\tan(\theta/2) - \mu}{\sigma}\right)\right)\right]^{-2} \exp\left(-\left(\frac{\tan(\theta/2) - \mu}{\sigma}\right)\right),$$

where $\sigma > 0$, $-\pi \leq \theta < \pi$ equivalently

$$g(\theta) = \frac{1}{8\sigma} \sec^2\left(\frac{\theta}{2}\right) \operatorname{sech}^2\left[\frac{\tan(\theta/2)}{2\sigma}\right].$$

Differentiating $g(\theta)$ with respect to θ we get

$$g'(\theta) = \frac{1}{8\sigma} \sec^2\left(\frac{\theta}{2}\right) \operatorname{sech}^2\left(\frac{\tan(\theta/2)}{2\sigma}\right) \left[\tan\left(\frac{\theta}{2}\right) - \frac{1}{2\sigma} \sec^2\left(\frac{\theta}{2}\right) \tanh\left(\frac{\tan(\theta/2)}{2\sigma}\right)\right].$$

For stationary points $g'(\theta) = 0 \Rightarrow \tanh(\tan(\theta/2)/(2\sigma)) - \sigma \sin\theta = 0 \Rightarrow \theta = 0, 2\pi, 4\pi, \ldots,$ are the stationary points. $\theta = 0$ in the only stationary point which lies in the domain of $g(\theta)$.

Differentiating $g'(\theta)$ with respect to θ, we get at $\theta = 0$

$$g'(0) = \frac{4\sigma^2 - 1}{64\sigma^3}.$$

$g(\theta)$ has maximum value at $\theta = 0$ if and only if $g'(0) < 0 \Leftrightarrow \sigma < 0.5 \quad (\sigma > 0)$.
$g(\theta)$ has minimum value at $\theta = 0$ if and only if $g'(0) > 0 \Leftrightarrow \sigma > 0.5 \quad (\sigma > 0)$.
Hence Stereographic logistic distribution is unimodal if $\sigma < 0.5$ and bimodal if $\sigma > 0.5$. ■

Graphs of probability density function of Stereographic Logistic Distribution for various values of σ and $\mu = 0$ are presented here.

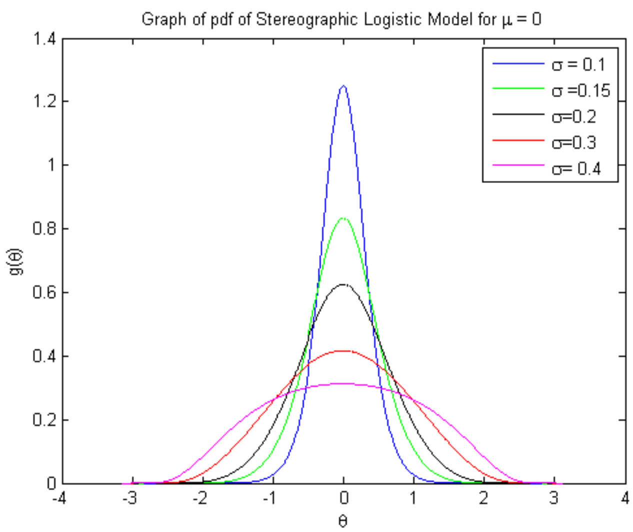

Figure 1. Graph of pdf of Stereographic Logistic model for $\mu = 0$

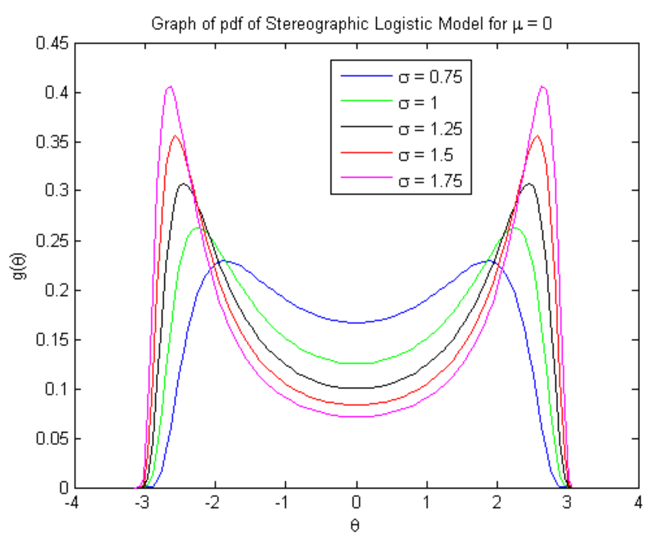

Figure 2. Graph of pdf of Stereographic Logistic model for $\mu = 0$

4. The Characteristic function of Stereographic Model

The Characteristic function of a Circular model with probability density function $g(\theta)$ is defined as $\varphi_p(\theta) = \int_0^{2\pi} e^{ip\theta} g(\theta) d\theta$, $p \in \mathbb{Z}$. Ramabhadra Sarma et al. (2009, 2011) derived the characteristic functions of some new wrapped models based on the Proposition (Jammalamadaka and Sengupta, 2001, p. 31). This proposition cannot be applied directly in case of Stereographic Circular Model. The characteristic function of a Stereographic Circular model can be obtained in terms of respective linear model. Lukacs and Laha (1970) proved the following theorem related to the characteristic function of linear model which is applied here in the case of Stereographic Circular Models.

THEOREM 4.1 Let X be a random variable with distribution function $F(x)$ and suppose that $S(x)$ is a finite and single-valued function of x. The characteristic function of $f_Y(t)$ of the random variable $Y = S(x)$ is then given by $f_Y(t) = \mathrm{E}(e^{itY}) = \mathrm{E}(e^{itS(X)}) = \int_{-\infty}^{\infty} e^{itS(X)} dF(x)$. By applying the above theorem we derive the characteristic function of a Stereographic Circular model.

THEOREM 4.2 (Phani et al., 2012a) If $G(\theta)$ and $g(\theta)$ are the cdf and pdf of the Stereographic Circular model and $F(x)$ and $f(x)$ are cdf and pdf of the respective linear model, then characteristic function of Stereographic Model is $\varphi_{X_S}(p) = \varphi_{2\tan^{-1}(x/\nu)}(p)$, $p \in \mathrm{Z}$.

PROOF

$$\begin{aligned}
\varphi_{X_S}(p) &= \int_{-\pi}^{\pi} e^{ip\theta} d(G(\theta)), \quad p \in \mathrm{Z} \\
&= \int_{-\pi}^{\pi} e^{ip\theta} d(F(\nu \tan(\theta/2))) \\
&= \int_{-\infty}^{\infty} e^{ip(2\tan^{-1}(x/\nu))} f(x) dx, \quad \text{taking } x = \nu \tan(\theta/2) \\
&= \varphi_{2\tan^{-1}(x/\nu)}(x).
\end{aligned}$$

■

The Characteristic function of Stereograph Logistic Model:

$$\begin{aligned}
\Phi_{X_S}(p) &= \int_{-\pi}^{\pi} e^{ip\theta} g(\theta) d\theta, \\
&= \int_{-\infty}^{\infty} e^{ip(2\tan^{-1}(x/\nu))} \frac{1}{\sigma} \left[1 + e^{-(x/\sigma)}\right]^{-2} e^{-(x/\sigma)} dx, \\
&= \frac{2}{\sigma} \int_{0}^{\infty} \cos p(2\tan^{-1}(x/\nu)) \left[1 + e^{-(x/\sigma)}\right]^{-2} e^{-(x/\sigma)} dx \quad (\text{since } f(x) \text{ is even}).
\end{aligned}$$

As the integral cannot be obtained analytically, MATLAB techniques are applied for the evaluation of the values of the characteristic function.

4.1 Trigonometric moments of the Stereographic Logistic Model

We also assume here that $\mu = 0$ in density function of SLG model. The trigonometric moments of the distribution are given by $\{\varphi_p : p = \pm1, \pm2, \pm3, \ldots\}$, where $\varphi_p = \alpha_p + i\beta_p$, with $\alpha_p = \mathrm{E}(\cos p\theta)$ and $\beta_p = \mathrm{E}(\sin p\theta)$ being the p^{th} order cosine and sine moments of the random angle θ, respectively. Because the Stereographic Logistic distribution is symmetric about $\mu = 0$, it follows that the sine moments are zero. Thus, $\varphi_p = \alpha_p$.

THEOREM 4.3 Under the pdf of the Stereographic Logistic distribution with $\mu = 0$, the first two $\alpha_p = \mathrm{E}(\cos p\theta)$, $p = 1, 2$ ar given as follows

$$\alpha_1 = 1 - \frac{2}{\sigma\sqrt{\pi}} \sum_{i=1}^{\infty} (-1)^{n-1} n G_{13}^{31} \left(\frac{n^2}{4\sigma^2} \middle|_{-\frac{1}{2},0,\frac{1}{2}}^{-\frac{1}{2}} \right),$$

$$\alpha_2 = 1 + \frac{8}{\sigma\sqrt{\pi}} \sum_{i=1}^{\infty} (-1)^{n-1} n \left\{ G_{13}^{31} \left(\frac{n^2}{4\sigma^2} \middle|_{\frac{-1}{2},0,\frac{1}{2}}^{-\frac{3}{2}} \right) - G_{13}^{31} \left(\frac{n^2}{4\sigma^2} \middle|_{-\frac{1}{2},0,\frac{1}{2}}^{-\frac{1}{2}} \right) \right\},$$

where

$$\int_0^{\infty} x^{2\nu-1} (u^2 + x^2)^{Q-1} e^{\mu x} dx = \frac{u^{2\nu+2Q-2}}{2\sqrt{\pi}\Gamma(1-Q)} G_{13}^{31} \left(\frac{\mu^2 u^2}{4} \middle|_{1-Q-\nu,0,\frac{1}{2}}^{1-\nu} \right),$$

for $|\arg u\pi| < \pi/2$, $\mathrm{Re}\,\mu > 0$ and $\mathrm{Re}\,\nu > 0$ and $G_{13}^{31} \left(\frac{\mu^2 u^2}{4} \middle|_{1-Q-\nu,0,\frac{1}{2}}^{1-\nu} \right)$ is called as Meijer's G-function (Gradshteyn and Ryzhik, 2007, Formula 3.389.2).

PROOF $\varphi_p = \displaystyle\int_{-\pi}^{\pi} \cos(p\theta) g(\theta) d\theta = \alpha_p$, where

$$\alpha_p = \frac{1}{2\sigma} \int_{-\pi}^{\pi} \cos(p\theta) \sec^2\left(\frac{\theta}{2}\right) e^{-\frac{1}{\sigma}\tan(\theta/2)} \left\{1 + e^{-\frac{1}{\sigma}\tan(\theta/2)}\right\}^{-2} d\theta.$$

We derive the first order trigonometric moment

$$\alpha_1 = \frac{1}{2\sigma} \int_{-\pi}^{\pi} \cos\theta \sec^2\left(\frac{\theta}{2}\right) e^{-\frac{1}{\sigma}\tan(\theta/2)} \left\{1 + e^{-\frac{1}{\sigma}\tan(\theta/2)}\right\}^{-2} d\theta,$$

as follows.

Consider the transformation $x = \tan(\theta/2)$, $\cos\theta = 1 - \frac{2x^2}{1+x^2}$ and the above formula

$$\begin{aligned} \alpha_1 &= \frac{2}{\sigma} \int_0^{\infty} \left[1 - \frac{2x^2}{1+x^2}\right] e^{-x/\sigma} \left\{1 + e^{-x/\sigma}\right\}^{-2} dx, \\ &= 1 - \frac{4}{\sigma} \int_0^{\infty} x^2 (1+x^2)^{-1} \sum_{n=1}^{\infty} (-1)^{n-1} n e^{-nx/\sigma} dx \\ \alpha_1 &= 1 - \frac{2}{\sigma\sqrt{\pi}} \sum_{i=1}^{\infty} (-1)^{n-1} n G_{13}^{31} \left(\frac{n^2}{4\sigma^2} \middle|_{-\frac{1}{2},0,\frac{1}{2}}^{-\frac{1}{2}} \right). \end{aligned}$$

We derive the second cosine moment

$$\alpha_2 = \frac{1}{2\sigma} \int_{-\pi}^{\pi} \cos\theta \sec^2\left(\frac{\theta}{2}\right) e^{-\frac{1}{\sigma}\tan(\theta/2)} \left\{1 + e^{-\frac{1}{\sigma}\tan(\theta/2)}\right\}^{-2} d\theta$$

as follows.

Consider the transformation $x = \tan(\theta/2)$, $\cos 2\theta = 1 + \frac{8x^4}{(1+x^2)^2} - \frac{8x^2}{(1+x^2)}$ and the above formula

$$\alpha_2 = \frac{2}{\sigma}\int_0^\infty \left[1 + \frac{8x^4}{(1+x^2)^2} - \frac{8x^2}{(1+x^2)}\right] e^{-x/\sigma}\left\{1+e^{-x/\sigma}\right\}^{-2} dx,$$

$$= 1 + \frac{16}{\sigma}\int_0^\infty x^4(1+x^2)^{-2}\sum_{n=1}^{\infty}(-1)^{-1} n e^{-nx/\sigma} dx - \frac{16}{\sigma}\int_0^\infty x^2(1+x^2)^{-1}\sum_{n=1}^{\infty}(-1)^{-1} n e^{-nx/\sigma} dx$$

$$\alpha_2 = 1 + \frac{8}{\sigma\sqrt{\pi}}\sum_{i=1}^{\infty}(-1)^{n-1} n \left\{ G_{13}^{31}\left(\frac{n^2}{4\sigma^2}\Bigg|_{\frac{-1}{2},0,\frac{1}{2}}^{-\frac{3}{2}}\right) - G_{13}^{31}\left(\frac{n^2}{4\sigma^2}\Bigg|_{-\frac{1}{2},0,\frac{1}{2}}^{-\frac{1}{2}}\right)\right\}.$$

■

5. Application - Goodness-of fit for 50 noisy scrub birds Data

Data set: Orientation of 50 noisy scrub birds along the bank of a creck bed (Fisher, 1993, p. 252)

160°, 145°, 225°, 230°, 295°, 295°, 140°, 140°, 140°, 205°, 215°, 135°,
110°, 240°, 230°, 250°, 30°, 215°, 135°, 110°, 240°, 105°, 125°, 125°, 130°,
160°, 160°, 105°, 90°, 130°, 200°, 240°, 105°, 125°, 125°, 125°, 130°,
160°, 160°, 250°, 200°, 200°, 240°, 240°, 240°, 250°, 250°, 250°, 140°, 140°

The main goal is to provide for monitoring the orientations of the nest of noisy scrub birds, along the bank of a creek bed. The species of birds in this region are more important for the biodiversity. This study is important due to the growing evidence of the vulnerability of the noisy scrub birds to chance in biodiversity. To demonstrate the modeling behavior of Stereographic Logistic, Stereographic Double Exponential (Girija et al., 2014a) and Wrapped Logistic models (Dattatreya Rao et al., 2007), we verify goodness-of-fit for noisy scrub birds data of size $n = 50$. The data plot is shown below

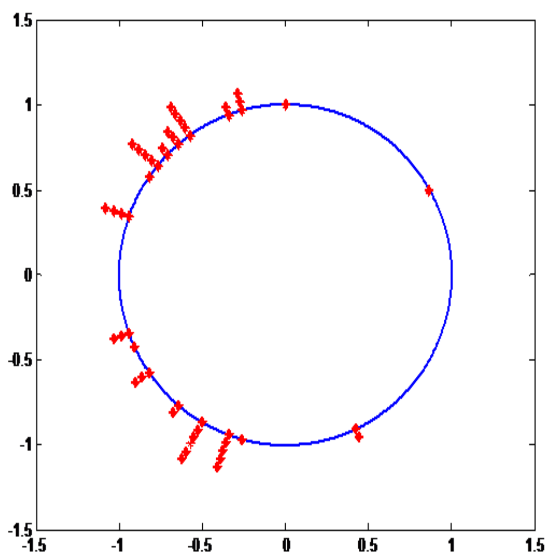

Figure 3. Graph of 50 Birds Data Plot

The pair of parameters can be estimated using several methods available in the literature of circular statistics. The two pairs of estimates $\hat{\mu} = -0.0695$, $\hat{\sigma} = 0.5374$ and $\tilde{\mu} = -0.3491$, $\tilde{\sigma} = 0.5165$ respectively.

We consider Stereographic Logistic, Stereographic Double Exponential (Girija et al., 2014a) and Wrapped Logistic models (Dattatreya Rao et al., 2007) to verify goodness-of-fit for noisy scrub birds data of size $n = 50$. As this is a large sample, we apply Kuiper's and modified Watson's U^2 tests. The statistic of Watson's U^2 test for large sample (Mardia and Jupp, 2000) is $W^2 = V_n^2/\pi^2$ where V_n the statistic of Kuipers test.

The statistics of the two goodness-of-fit tests viz., Kuipers and Watsons U^2 tests are computed for the said circular models and are tabulated.

Table 1. Statistics of the Kuiper's and Watson's U^2 tests of goodness-of-fit tests for Stereographic Logistic Distribution (SLD) , Stereographic Double Exponential Distribution (SDED) and Wrapped Logistic Distribution (WLD).

	SLD		SDED		WLD	
Sample size $n = 50$	$\hat{\mu}, \hat{\sigma}$	$\tilde{\mu}, \tilde{\sigma}$	$\hat{\mu}, \hat{\sigma}$	$\tilde{\mu}, \tilde{\sigma}$	$\hat{\mu}, \hat{\sigma}$	$\tilde{\mu}, \tilde{\sigma}$
Kuiper's Test	5.4914	5.4497	6.0315	5.9857	6.2367	6.2656
Watson's U^2 Test	0.0580	0.0571	0.0699	0.0689	0.0748	0.0755

On the lines of algorithm in Devaraaj (2012) the cut off points for the data set of sample size $n = 50$ are computed using MATLAB techniques (Amos, 2014).

Table 2. The cut of points for the data set of sample size $n = 50$

LOS Tests	1%	5%	10%
Kuiper's Test	$0.6891 - 2.0612$	$0.8088 - 1.8927$	$0.8457 - 1.7329$
Watson's U^2 Test	$0.0177 - 0.2960$	$0.0209 - 0.2163$	$0.0265 - 0.1849$

On the basis of cut off points for the data set of sample size $n = 50$, the statistics computed above reveals that the Data Set follows all the three circular models viz., Stereographic Logistic, Stereographic Double Exponential and Wrapped Logistic models at all the levels of significance (i.e., 1%, 5%, and 10%) by modified Watson's U^2 test , for two sets of estimated parameters $\hat{\mu} = -0.0695$, $\hat{\sigma} = 0.5374$ and $\tilde{\mu} = -0.3491$, $\tilde{\sigma} = 0.5165$.

For a given data, in a case more than one parametric circular model fits well, then choosing the appropriate model which is the best fit, can be decided by the criteria Maximum Log Likelihood (MLL), Akaikes Information Criteria (AIC) and Bayesian Information Criteria (BIC). The computations of measures of relative performances are presented.

Table 3. - Measures of Relative Performance for Goodnessof-fit at $\hat{\mu} = -0.0695$, $\hat{\sigma} = 0.5374$ and $\tilde{\mu} = -0.3491$, $\tilde{\sigma} = 0.5165$ for Data Set

Distribution	Estimates of the parameters	MLL	AIC	BIC
WLD	$\hat{u} = -0.0695, \hat{\sigma} = 0.5374$	-171.9734	175.9734	351.7708
	$\tilde{u} = -0.3491, \tilde{\sigma} = 0.5165$	-178.0112	182.0112	363.8465
SLD	$\hat{u} = -0.0695, \hat{\sigma} = 0.5374$	-149.6020	153.6020	307.0280
	$\tilde{u} = -0.3491, \tilde{\sigma} = 0.5165$	-156.5730	160.5730	320.9701
SDED	$\hat{u} = -0.0695, \hat{\sigma} = 0.5374$	-188.8666	192.8666	385.5572
	$\tilde{u} = -0.3491, \tilde{\sigma} = 0.5165$	-177.3916	181.3916	362.6073

On the basis of AIC, BIC and MLL we identify that Stereographic Logistic model is the superior fit than Stereographic Double Exponential and Wrapped Logistic models for

$\hat{\mu} = -0.0695$, $\hat{\sigma} = 0.5374$. Also it is observed that W^2_{50} for Stereographic Logistic model at $\tilde{\mu} = -0.3491$, $\tilde{\sigma} = 0.5165$ is the smallest of all the values of Watsons U^2 test.

6. Concluding Remarks

Introducing Stereographic Logistic model, it is shown that the proposed new model is a good fit for a live data set of 50 noisy scrub birds based on Watson's U^2 and Kuiper tests. The characteristic function of the proposed model is found to be not in a closed form and hence first two trigonometric moments are derived. Referring to the literature, it was noticed that Wrapped Logistic and Stereographic Double Exponential models are also found to be good fits. Hence the relative performance of the proposed new model viz Stereographic Logistic model with respect to the other two models mentioned is studied based on MLL, AIC and BIC and was established that the Stereographic Logistic model is a superior model in fitting the said live data. The graphs of the pdf for various combinations of parameters are presented.

Wrapped versions of circular models are almost not in closed form whereas Stereographic circular models exhibit closed form. Computations are involved in the methodology of inverse stereographic projection and can be done using MATLAB (Amos, 2014). In wrapping the resultant model is always a circular model whereas the models obtained by inducing inverse stereographic projection will be circular/semicircular depending on the domain of linear random variable.

The above finer pointer will become a motivating factor to work on l-axial (arc) models, and they can be viewed as the most general angular models from which Circular, Semicircular and other kinds of models could be deduced as particular cases.

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