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On the improvement of product method of estimation in ranked set sampling

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Abstract

Many forms of ranked set samples have been introduced recently for estimating the population mean and other parameters. For estimating a finite population mean in ranked set sampling under product method of estimation, simple linear transformations using the known coefficient of skewness, coefficient of kurtosis and standard deviation of the auxiliary variable have been considered in this paper. It has been shown that this method is highly beneficial to the estimation based on Simple Random Sampling (SRS). The bias and mean squared error of the suggested estimators with large sample approximation are derived. Theoretically, it is shown that these suggested estimators are more efficient than the estimators in simple random sampling. A numerical example is also carried out to demonstrate the merits of the proposed estimators using RSS over the usual estimators in SRS.

Keywords: Ranked set sampling · product estimators · auxiliary variable · population mean · coefficient of variation · coefficient of kurtosis · Standard deviation · bias · mean squared error

Mathematics Subject Classification: Primary 62D05

1. Introduction

Product method of estimation is well-known technique for estimating the population mean of a study character when population mean of an auxiliary character is known and it is negatively correlated with study character. Sisodia and Dwivedi (1981) used the known CV of the auxiliary character for estimating population mean of a study character in ratio method of estimation. Following the work of Sisodia and Dwivedi (1981), Pandey and Dubey (1988) proposed a modified product estimator for population mean of a study character using known CV of an auxiliary character. Recently Upadhyaya and Singh (1999) proposed new product estimators using known CV and coefficient of kurtosis of an auxiliary character. For estimating a finite population mean under product method of estimation, simple linear transformations using the known coefficient of skewness, coefficient of kurtosis and standard deviation of the auxiliary variable have been considered by Singh (2003).

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Ranked set sampling describes a great variety of techniques for using auxiliary information to obtain more efficient estimators. McIntyre (1952) was the first to suggest using Ranked set sampling (RSS) instead of Simple random sampling (SRS) to increase the efficiency of estimator of population mean and developed by Takahasi and Wakimoto (1968) used to estimate the ratio. They proved that the sample mean obtained by using the RSS is an unbiased estimator of the population mean and has smaller variance than the sample mean obtained by using the SRS of the same sample size. Swami and Muttlak (1996) used ranked set sampling to define the ratio estimators for population mean. Bouza (2008) developed study of the estimation of the population mean using of product estimator under ranked set. Kadilar et al. (2009) used this technique to improve ratio estimator given by Prasad (1989).

The traditional product estimator given by Murthy (1964) for estimating the population mean $\overline{Y}$ is defined as

$$\overline{y}_P = \overline{y}\left(\frac{\overline{x}}{\overline{X}}\right) \tag{1}$$

When the population coefficient of variation of variate C_x is known, Pandey and Dubey (1988) considered the modified product estimator of Y as

$$\overline{y}_{MP} = \overline{y}\left(\frac{\overline{x}+C_x}{\overline{X}+C_x}\right) \tag{2}$$

Utilizing the information on co-efficient of variation C_x and co-efficient of kurtosis $\beta_2(x)$, Upadhyaya and Singh (1999) suggested the two more modified product estimators which are reproduced below

$$\overline{y}_{us1} = \overline{y}\left(\frac{\overline{x}C_x+\beta_2(x)}{\overline{X}C_x+\beta_2(x)}\right) \tag{3}$$

$$\overline{y}_{us2} = \overline{y}\left(\frac{\overline{x}\beta_2(x)+C_x}{\overline{X}\beta_2(x)+C_x}\right) \tag{4}$$

Motivated with the above arguments and utilizing the known values of σ_x, $\beta_1(x)$ and $\beta_2(x)$, Singh (2003) suggested the following three more reasonable transformations for x and corresponding estimators for $\overline{Y}$, which are as follows

$$\overline{T}_{P1} = \overline{y}\left(\frac{\overline{x}+\sigma_x}{\overline{X}+\sigma_x}\right) \tag{5}$$

$$\overline{T}_{P2} = \overline{y}\left(\frac{\overline{x}\beta_1(x)+\sigma_x}{\overline{X}\beta_1(x)+\sigma_x}\right) \tag{6}$$

$$\overline{T}_{P3} = \overline{y}\left(\frac{\overline{x}\beta_2(x)+\sigma_x}{\overline{X}\beta_2(x)+\sigma_x}\right) \tag{7}$$

To the first degree of approximation the mean squared error (MSE) of the estimators

$\overline{y}_P$, $\overline{y}_{MP}$, $\overline{y}_{us1}$, $\overline{y}_{us2}$, $\overline{T}_{P1}$, $\overline{T}_{P2}$ and $\overline{T}_{P3}$ respectively are

$$\text{MSE}(\overline{y}_P) = \theta\overline{Y}^2(C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x) \tag{8}$$

$$\text{MSE}(\overline{y}_{MP}) = \theta\overline{Y}^2(C_y^2 + \delta^2C_x^2 + 2\delta\rho_{yx}C_yC_x) \tag{9}$$

$$\text{MSE}(\overline{y}_{us1}) = \theta\overline{Y}^2(C_y^2 + \gamma_1^2C_x^2 + 2\gamma_1\rho_{yx}C_yC_x) \tag{10}$$

$$\text{MSE}(\overline{y}_{us2}) = \theta\overline{Y}^2(C_y^2 + \gamma_2^2C_x^2 + 2\gamma_2\rho_{yx}C_yC_x) \tag{11}$$

$$\text{MSE}(\overline{T}_{P1}) = \theta\overline{Y}^2(C_y^2 + \phi_1^2C_x^2 + 2\phi_1\rho_{yx}C_yC_x) \tag{12}$$

$$\text{MSE}(\overline{T}_{P2}) = \theta\overline{Y}^2(C_y^2 + \phi_2^2C_x^2 + 2\phi_2\rho_{yx}C_yC_x) \tag{13}$$

$$\text{MSE}(\overline{T}_{P3}) = \theta\overline{Y}^2(C_y^2 + \phi_3^2C_x^2 + 2\phi_3\rho_{yx}C_yC_x) \tag{14}$$

where $\theta = 1/n$ (on ignoring $f = n/N$),

$$C_y^2 = \frac{S_y^2}{\overline{Y}^2}, \qquad C_x^2 = \frac{S_x^2}{\overline{X}^2}, \qquad C_{yx} = \frac{S_{ys}}{\overline{X}\overline{Y}} = \rho_{yx}C_yC_x,$$

here C_y and C_x are coefficient of variations of y and x respectively and ρ is correlation coefficient.

Here

$$\delta = \frac{\overline{X}}{\overline{X} + C_x}, \qquad \gamma_1 = \frac{\overline{X}C_x}{\overline{X}C_x + \beta_2(x)}, \qquad \gamma_2 = \frac{\overline{X}\beta_2(x)}{\overline{X}\beta_2(x) + C_x},$$

$$\phi_1 = \frac{\overline{X}}{\overline{X} + \sigma_x}, \qquad \phi_2 = \frac{\overline{X}\beta_1(x)}{\overline{X}\beta_1(x) + \sigma_x}, \qquad \phi_3 = \frac{\overline{X}\beta_2(x)}{\overline{X}\beta_2(x) + \sigma_x},$$

$$S_y^2 = \frac{\sum_{i=1}^N (Y_i - \overline{Y})^2}{N-1}, \qquad S_x^2 = \frac{\sum_{i=1}^N (X_i - \overline{X})^2}{N-1} \quad \text{and} \quad S_{yx} = \frac{\sum_{i=1}^N (Y_i - \overline{Y})(X_i - \overline{X})}{N-1}.$$

2. Product Estimator in ranked set sampling

In Ranked set sampling (RSS), m independent random sets are chosen, each of size m and unit in each set are selected with equal probability and without replacement from the population. The members of each random set are ranked with respect to the characteristic of the study variable or auxiliary variable. Then, the smallest unit is selected from the first ordered set and the second smallest unit is selected from the second ordered set. By this way, this procedure is continued until the unit with the largest rank is chosen from the m^{th} set. This cycle may be repeated r times, so $mr(= n)$ units have been measured during this process.

When we rank on the auxiliary variable, let $(Y_{[i]}, X_{(i)})$ denote a i^{th} judgment ordering in the i^{th} set for the study variable and i^{th} set for the auxiliary variable.

Bouza (2008) defined the product estimator for the population mean in ranked set sampling as

$$\overline{y}_{P,RSS} = \overline{y}_{[n]}\left(\frac{\overline{x}_{(n)}}{\overline{X}}\right) \tag{15}$$

where

$$\overline{y}_{[n]} = \frac{1}{n}\sum_{i=1}^{n} y_{[i]} \quad \text{and} \quad \overline{x}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$$

are the ranked set sample means for variables y and x respectively.

To the first degree of approximation the mean squared error (MSE) of the estimator $\overline{y}_{P,\,RSS}$ is

$$\text{MSE}(\overline{y}_{P,\,RSS}) = \overline{Y}^2[\theta(C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x) - (W_{y[i]} + W_{y(i)})^2] \tag{16}$$

where

$$\theta = \frac{1}{mr}, \qquad C_y^2 = \frac{S_y^2}{\overline{Y}^2}, \qquad C_x^2 = \frac{S_x^2}{\overline{X}^2}, \qquad C_{yx} = \frac{S_{ys}}{\overline{X}\,\overline{Y}} = \rho_{yx}C_yC_x,$$
$$W_{x(i)}^2 = \frac{\sum_{i=1}^{m}\tau_{x(i)}^2}{m^2r\overline{X}^2}, \qquad W_{y[i]}^2 = \frac{\sum_{i=1}^{m}\tau_{y[i]}^2}{m^2r\overline{Y}^2} \qquad \text{and} \quad W_{yx(i)} = \frac{\sum_{i=1}^{m}\tau_{yx(i)}}{m^2r\overline{Y}\,\overline{X}}.$$

Here we would also like to remind that

$$\tau_{x(i)} = \mu_{x(i)} - \overline{X}, \qquad \tau_{y[i]} = \mu_{y[i]} - \overline{Y} \quad \text{and} \quad \tau_{yx(i)} = (\mu_{x(i)} - \overline{X})(\mu_{y[i]} - \overline{Y}).$$

3. Suggested estimators based on ranked set sampling

Motivated by Pandey and Dubey (1988), we suggest product type estimator for $\overline{Y}$ using RSS, when the population coefficient of variation of auxiliary variable C_x is known as

$$\overline{y}_{MM1,\,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)} + C_x}{\overline{X} + C_x}\right] \tag{17}$$

The Bias and MSE of the estimator $\overline{y}_{MM1,\,RSS}$ can be given by

$$\implies B(\overline{y}_{MM1,\,RSS}) = \overline{Y}\left[\delta(\theta\rho_{yx}C_yC_x - W_{yx(i)})\right] \tag{18}$$

$$\implies \text{MSE}(\overline{y}_{MM1,\,RSS}) = \overline{Y}^2\left[\theta(C_y^2 + C_x^2 + 2\delta\rho_{yx}C_yC_x) - (W_{y[i]} + \delta W_{x(i)})^2\right] \tag{19}$$

Motivated by Upadhyaya and Singh (1999), we suggest the product type estimators considered both coefficients of variation and Kurtosis in ranked set sampling as

$$\overline{y}_{MM2,\,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)}C_x + \beta_2(x)}{\overline{X}C_x + \beta_2(x)}\right] \tag{20}$$

$$\overline{y}_{MM3,\,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)}\beta_2(x) + C_x}{\overline{X}\beta_2(x) + C_x}\right] \tag{21}$$

The Bias and MSE of the estimator $\overline{y}_{MM2,\,RSS}$ can be given by

$$B(\overline{y}_{MM2,\,RSS}) = E(\overline{y}_{MM2,\,RSS}) - \overline{Y}$$

Here $\overline{y}_{MM2,\,RSS} = \overline{Y}(1+\epsilon_0)(1+\gamma_1\epsilon_1)$, where $\gamma_1 = \frac{\overline{X}C_x}{\overline{X}C_x+\beta_2(x)}$

$$B(\overline{y}_{MM2,\,RSS}) = \overline{Y}[\gamma_1 E(\epsilon_0\epsilon_1)]$$

because $E(\epsilon_0) = E(\epsilon_1) = 0$

$$\implies B(\overline{y}_{MM2,\,RSS}) = \overline{Y}\left[\gamma_1(\theta\rho_{yx}C_yC_x - W_{yx(i)})\right]$$

Now

$$\begin{aligned}
\text{MSE}(\overline{y}_{MM2,\,RSS}) &= E(\overline{y}_{MM2,\,RSS} - \overline{Y})^2 \\
&= \overline{Y}^2 E[\epsilon_0 + \gamma_1\epsilon_1 + 2\gamma_1\epsilon_0\epsilon_1]^2 \\
&= \overline{Y}^2 E[\epsilon_0^2 + \gamma_1^2\epsilon_1^2 + 2\gamma_1\epsilon_0\epsilon_1] \\
&= \overline{Y}^2[\theta C_y^2 - W_{y[i]}^2 + \gamma_1^2(\theta C_x^2 - W_{x(i)}^2) + 2\gamma_1(\theta\rho_{yx}C_yC_x - W_{yx(i)}) \\
\implies \text{MSE}(\overline{y}_{MM2,\,RSS}) &= \overline{Y}^2[\theta(C_y^2 + \gamma_1^2C_x^2 + 2\gamma_1\rho_{yx}C_yC_x) - (W_{y[i]} + \gamma_1 W_{x(i)})^2] \qquad (22)
\end{aligned}$$

Similarly bias and mean squared error of the estimator $\overline{y}_{MM3,\,RSS}$ can be obtained respectively by changing the place of coefficient of kurtosis and coefficient of variation as

$$B(\overline{y}_{MM3,\,RSS}) = \overline{Y}[\gamma_2(\theta\rho_{yx}C_yC_x - W_{yx(i)})] \qquad (23)$$

$$\text{MSE}(\overline{y}_{MM3,\,RSS}) = \overline{Y}^2\left[\theta(C_y^2 + \gamma_2^2C_x^2 + 2\gamma_2\rho_{yx}C_yC_x) - (W_{y[i]} + \gamma_2 W_{x(i)})^2\right] \qquad (24)$$

Adapting the estimators in (5), (6) and (7), given by Singh (2003) and utilizing the known values of σ_x, $\beta_1(x)$ and $\beta_2(x)$, we propose the following three more reasonable transformations for x and corresponding estimators for $\overline{Y}$, which are as follows.

Let $\overline{z}_{i(n)} = \alpha_i\overline{x}_{(n)} + \sigma_x$ is the ranked set sample mean of the transformed variable z and $\overline{Z}_i = \alpha_i\overline{X} + \sigma_x$ is the corresponding population mean, where $\alpha_1 = 1$, $\alpha_2 = \beta_1(x)$ and $\alpha_3 = \beta_2(x)$ are the known values of α_i's for $i = 1,\,2,\,3$ Subsequently, the following three new modified product estimators of $\overline{Y}$ using ranked set sampling are considered as

$$\overline{y}_{MMp1,\,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)} + \sigma_x}{\overline{X} + \sigma_x}\right] = \frac{\overline{y}_{[n]}\overline{z}_{1(n)}}{\overline{Z}_1},\ \text{for } i = 1, \quad \alpha_1 = 1 \qquad (25)$$

$$\overline{y}_{MMp2,\,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)}\beta_1(x) + \sigma_x}{\overline{X}\beta_1(x) + \sigma_x}\right] = \frac{\overline{y}_{[n]}\overline{z}_{2(n)}}{\overline{Z}_2},\ \text{for } i = 2, \quad \alpha_2 = \beta_1(x) \qquad (26)$$

$$\overline{y}_{MMp3,\,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)}\beta_2(x) + \sigma_x}{\overline{X}\beta_2(x) + \sigma_x}\right] = \frac{\overline{y}_{[n]}\overline{z}_{3(n)}}{\overline{Z}_3},\ \text{for } i = 3, \quad \alpha_3 = \beta_2(x) \qquad (27)$$

Estimators proposed in (25) to (27) can be re-written as a sequence of new modified product estimators in the following form

$$\overline{y}_{MMpi,\,RSS} = \overline{y}_{[n]}\left[\frac{\alpha_i\overline{x}_{(n)} + \sigma_x}{\alpha_i\overline{X} + \sigma_x}\right] = \frac{\overline{y}_{[n]}\overline{z}_{i(n)}}{\overline{Z}_i}, \qquad (i = 1,\,2,\,3) \qquad (28)$$

The Bias and Mean Square Error (MSE) of the Proposed Sequence of Estimators,

$\overline{y}_{MMpi,\,RSS}$ $(i = 1,\,2,\,3)$ can be found as follow

$$\overline{y}_{MMpi,\,RSS} = \overline{Y}(1+\epsilon_0)(1+\phi_i\epsilon_1), \quad (i = 1,\,2,\,3) \tag{29}$$

where

$$\phi_1 = \frac{\overline{X}}{\overline{X}+\sigma_x}, \qquad \phi_2 = \frac{\overline{X}\beta_1(x)}{\overline{X}\beta_1(x)+\sigma_x}, \qquad \phi_3 = \frac{\overline{X}\beta_2(x)}{\overline{X}\beta_2(x)+\sigma_x}$$

For $i = 1,\,2,\,3$ and $\phi_i < 1$, now expanding the terms of (29) and taking expectations, the bias and mean square error of the proposed sequence of estimators $\overline{y}_{MMpi,\,RSS}$, $(i = 1,\,2,\,3)$ are given by

$$B(\overline{y}_{MMpi,\,RSS}) = E(\overline{y}_{MMpi,\,RSS}) - \overline{Y} = \overline{Y}\left[\phi_i(\theta\rho_{yx}C_yC_x - W_{yx(i)})\right] \tag{30}$$

$$\begin{aligned}\text{MSE}(\overline{y}_{MMpi,\,RSS}) &= E(\overline{y}_{MMpi,\,RSS} - \overline{Y})^2 \\ &= \overline{Y}^2[\theta(C_y^2 + \phi_i^2C_x^2 + 2\phi_i\rho_{yx}C_yC_x) - (W_{y[i]} + \phi_iW_{x(i)})^2]\end{aligned} \tag{31}$$

Bias and MSE of $\overline{y}_{MMp1,\,RSS}$, $\overline{y}_{MMp2,\,RSS}$ and $\overline{y}_{MMp3,\,RSS}$ defined in (25) to (27) can be obtained by substituting the values of ϕ_i, $(i = 1,\,2,\,3)$ in (30) and (31), respectively as

$$B(\overline{y}_{MMp1,\,RSS}) = \overline{Y}\left[\phi_1(\theta\rho_{yx}C_yC_x - W_{yx(i)})\right] \tag{32}$$

$$\text{MSE}(\overline{y}_{MMp1,\,RSS}) = \overline{Y}^2\left[\theta(C_y^2 + \phi_1^2C_x^2 + 2\phi_1\rho_{yx}C_yC_x) - (W_{y[i]} + \phi_1W_{x(i)})^2\right] \tag{33}$$

$$B(\overline{y}_{MMp2,\,RSS}) = \overline{Y}\left[\phi_2(\theta\rho_{yx}C_yC_x - W_{yx(i)})\right] \tag{34}$$

$$\text{MSE}(\overline{y}_{MMp2,\,RSS}) = \overline{Y}^2\left[\theta(C_y^2 + \phi_2^2C_x^2 + 2\phi_2\rho_{yx}C_yC_x) - (W_{y[i]} + \phi_2W_{x(i)})^2\right] \tag{35}$$

$$B(\overline{y}_{MMp3,\,RSS}) = \overline{Y}\left[\phi_3(\theta\rho_{yx}C_yC_x - W_{yx(i)})\right] \tag{36}$$

$$\text{MSE}(\overline{y}_{MMp3,\,RSS}) = \overline{Y}^2\left[\theta(C_y^2 + \phi_3^2C_x^2 + 2\phi_3\rho_{yx}C_yC_x) - (W_{y[i]} + \phi_3W_{x(i)})^2\right] \tag{37}$$

4. Efficiency Comparison

On comparing (9), (10), (11), (12), (13) and (14) with (19), (22), (24), (33), (35) and (37) respectively, we obtained

1) $\text{MSE}(\overline{y}_{MP}) - \text{MSE}(\overline{y}_{MM1,\,RSS}) = A_1 \geq 0$, where $A_1 = [W_{y[i]} + \delta W_{x(i)}]^2$

$$\Longrightarrow \text{MSE}(\overline{y}_{MP}) \geq \text{MSE}(\overline{y}_{MM1,\,RSS})$$

2) $\text{MSE}(\overline{y}_{us1}) - \text{MSE}(\overline{y}_{MM2,\,RSS}) = A_2 \geq 0$, where $A_2 = [W_{y[i]} + \gamma_1 W_{x(i)}]^2$

$$\Longrightarrow \text{MSE}(\overline{y}_{MP}) \geq \text{MSE}(\overline{y}_{MM1,\,RSS})$$

3) $\text{MSE}(\overline{y}_{us2}) - \text{MSE}(\overline{y}_{MM3,\,RSS}) = A_3 \geq 0$, where $A_3 = [W_{y[i]} + \gamma_2 W_{x(i)}]^2$

$$\Longrightarrow \text{MSE}(\overline{y}_{MP}) \geq \text{MSE}(\overline{y}_{MM1,\,RSS})$$

4) $\text{MSE}(\overline{T}_{P1}) - \text{MSE}(\overline{y}_{MMp1,\,RSS}) = A_4 \geq 0$, where $A_4 = [W_{y[i]} + \phi_1 W_{x(i)}]^2$

$$\Longrightarrow \text{MSE}(\bar{t}_{P1}) \geq \text{MSE}(\overline{y}_{MMp1,\,RSS})$$

5) $\text{MSE}(\overline{T}_{P2}) - \text{MSE}(\overline{y}_{MMp2,\,RSS}) = A_5 \geq 0$, where $A_5 = [W_{y[i]} + \phi_2 W_{x(i)}]^2$

$$\Longrightarrow \text{MSE}(\bar{t}_{P3}) \geq \text{MSE}(\overline{y}_{MMp1,\,RSS})$$

6) $\text{MSE}(\overline{T}_{P3}) - \text{MSE}(\overline{y}_{MMp3,\,RSS}) = A_6 \geq 0$, where $A_6 = [W_{y[i]} + \phi_3 W_{x(i)}]^2$

$$\Longrightarrow \text{MSE}(\overline{T}_{P3}) \geq \text{MSE}(\overline{y}_{MMp3,\,RSS})$$

It is easily seen that the MSE of the suggested estimators given in (17), (20), (21), (25), (26) and (27) are always smaller than the estimator given in (2) to (7) respectively, because A_1, A_2, A_3, A_4, A_5 and A_6 all are non-negative values. As a result, show that all the suggested new product estimators $\overline{y}_{MM1,\,RSS}$, $\overline{y}_{MM2,\,RSS}$, $\overline{y}_{MM3,\,RSS}$, $\overline{y}_{MMp1,\,RSS}$, $\overline{y}_{MMp2,\,RSS}$ and $\overline{y}_{MMp3,\,RSS}$ for the population mean using ranked set sampling are more efficient than the usual product type estimators $\overline{y}_{MP}$, $\overline{y}_{us1}$, $\overline{y}_{us2}$, $\overline{T}_{P1}$, $\overline{T}_{P2}$ and $\overline{T}_{P3}$.

5. Numerical Illustration

To compare efficiencies of various estimators of our study, here we have taken a population of size $N = 50$ on page 1111 (Appendix) from the book entitled *Advanced Sampling Theory with Applications*, Vol.2, by Sarjinder Singh published from Kluwer Academic Publishers. The example considers the data of Agricultural loans outstanding of all operating banks in different states of USA in 1997, where y is real estate farm loans (study variable) in \$000 and x is the non-real estate loans (auxiliary variable) in \$000.

For the above population, the parameters are summarized as below:

$$Y = 27771.73, \quad X = 43908.12,$$
$$\overline{Y} = 555.43, \quad \overline{X} = 878.16,$$
$$S_x^2 = 1176526, \quad S_y^2 = 342021.5,$$
$$C_x^2 = 1.5256, \quad C_y^2 = 1.1086,$$
$$\rho = 0.8038, \quad \beta_1(x) = 1.66 \quad \text{and} \quad \beta_2(x) = 1.9215$$

From the above population, we took 100 ranked set samples with size $m = 4$ and number of cycles, $r = 3$, so that $n(= mr) = 12$. For these 100 ranked set samples chosen, we have computed estimated MSE's of our different new estimators $\overline{y}_{MM1,\,RSS}$, $\overline{y}_{MM2,\,RSS}$, $\overline{y}_{MM3,\,RSS}$, $\overline{y}_{MMp1,\,RSS}$, $\overline{y}_{MMp2,\,RSS}$ and $\overline{y}_{MMp3,\,RSS}$ which are given in table 1. Table 2 shows MSE's of different estimators $\overline{y}_{MP}$, $\overline{y}_{us1}$, $\overline{y}_{us2}$, $\overline{T}_{P1}$, $\overline{T}_{P2}$ and $\overline{T}_{P3}$, given by Singh (2003).

Table 1. MSE of different product type estimators using SRS

Estimators	$\overline{y}_{MP}$	$\overline{y}_{us1}$	$\overline{y}_{us2}$	$\overline{T}_{P1}$	$\overline{T}_{P2}$	$\overline{T}_{P3}$
MSE	121272.33	121285.18	121379.99	60399.7	72215.29	75751.25

On comparing table 1 with table 2, we conclude that the Mean squared error of the suggested estimators are always smaller than the usual estimators given by author Singh (2003). As a result, show that all the suggested new product type estimators $\overline{y}_{MM1,\,RSS}$, $\overline{y}_{MM2,\,RSS}$, $\overline{y}_{MM3,\,RSS}$, $\overline{y}_{MMp1,\,RSS}$, $\overline{y}_{MMp2,\,RSS}$ and $\overline{y}_{MMp3,\,RSS}$ for the population mean using RSS are more efficient than the usual product type estimators $\overline{y}_{MP}$, $\overline{y}_{us1}$, $\overline{y}_{us2}$, $\overline{T}_{P1}$, $\overline{T}_{P2}$ and $\overline{T}_{P3}$ under SRS. Thus, if Coefficient of variation, Coefficient of Skewness, Coefficient of kurtosis and Standard Deviation are known of auxiliary variable x, these proposed estimators are recommended for use in practice.

6. Concluding Remarks

We have proposed new product type estimators for ranked set sampling on the lines of the estimators of Singh (2003) and obtained their MSE equations. By these equations, the MSE's of proposed estimators have been compared with corresponding simple random sampling estimators given by Singh (2003) and found that the proposed estimators have smaller MSE's than the corresponding estimators.These theoretical results have been supported by the above numerical example and thus it is concluded that all the suggested new product type estimators $\overline{y}_{MM1,\,RSS}$, $\overline{y}_{MM2,\,RSS}$, $\overline{y}_{MM3,\,RSS}$, $\overline{y}_{MMp1,\,RSS}$, $\overline{y}_{MMp2,\,RSS}$ and $\overline{y}_{MMp3,\,RSS}$ for the population mean using RSS are more efficient than the usual product type estimators $\overline{y}_{MP}$, $\overline{y}_{us1}$, $\overline{y}_{us2}$, $\overline{T}_{P1}$, $\overline{T}_{P2}$ and $\overline{T}_{P3}$ under SRS. Thus, if Coefficient of variation, Coefficient of Skewness, Coefficient of kurtosis and Standard Deviation are known of auxiliary variable x, these proposed estimators are recommended for use in practice.

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Appendix

In this section, we show how to obtain the bias and MSE of the proposed estimator $\overline{y}_{MM1,\,RSS}$.

Let $\overline{y}_{[n]} = \overline{Y}(1+\epsilon_0)$ and $\overline{x}_{(n)} = \overline{X}(1+\epsilon_1)$ so that $E(\epsilon_0) = E(\epsilon_1) = 0$

$$V(\epsilon_0) = E(\epsilon_0^2) = \frac{V(\overline{y}_{[n]})}{\overline{Y}^2} = \frac{[S_y^2 - \frac{\sum_{i=1}^{m}\tau_{y[i]}^2}{m}]}{mr\overline{Y}^2} = [\theta C_y^2 - W_{y[i]}^2]$$

similarly,

$$V(\epsilon_1) = E(\epsilon_1^2) = [\theta C_x^2 - W_{x(i)}^2]$$

and

$$Cov(\epsilon_0,\,\epsilon_1) = E(\epsilon_0,\,\epsilon_1) = \frac{Cov(\overline{y}_{[n]},\,\overline{x}_{(n)})}{\overline{XY}} = \frac{[S_{yx} - \frac{\sum_{i=1}^{m}\tau_{yx(i)}}{m}]}{mr\overline{Y}\,\overline{X}} = [\theta\rho_{yx}C_yC_x - W_{yx(i)}]$$

Further to validate first degree of approximation, we assume that the sample size is large enough to get $|\epsilon_0|$and $|\epsilon_1|$ as small so that the terms involving ϵ_0 and or ϵ_1 in a degree greater than two will be negligible.

Now

$$B(\overline{y}_{MM1,\,RSS}) = E(\overline{y}_{MM1,\,RSS}) - \overline{Y},$$

here $\overline{y}_{MM1,\,RSS} = \overline{Y}(1+\epsilon_0)(1+\delta\epsilon_1)$, where

$$\delta = \frac{\overline{X}}{\overline{X} + C_x}$$

$$B(\overline{y}_{MM1,\,RSS}) = \overline{Y}[\delta E(\epsilon_0\epsilon_1)],$$

because $E(\epsilon_0) = E(\epsilon_1) = 0$

$$\Longrightarrow \; B(\overline{y}_{MM1,\,RSS}) = \overline{Y}[\delta(\theta\rho_{yx}C_yC_x - W_{yx(i)})]$$

Now

$$\begin{aligned}
\text{MSE}(\overline{y}_{MM1,\,RSS}) &= E(\overline{y}_{MM1,\,RSS} - \overline{Y})^2 \\
&= \overline{Y}^2 E[\epsilon_0 + \delta\epsilon_1 + \delta\epsilon_0\epsilon_1]^2 \\
&= \overline{Y}^2 E[\epsilon_0^2 + \delta^2\epsilon_1^2 + 2\delta\epsilon_0\epsilon_1] \\
&= \overline{Y}^2[\theta C_y^2 - W_{y[i]}^2 + \delta^2(\theta C_x^2 - W_{x(i)}^2) + 2\delta(\theta\rho_{yx}C_yC_x - W_{yx(i)})] \\
&= \overline{Y}^2[\theta(C_y^2 + \delta^2C_x^2 + 2\delta\rho_{yx}C_yC_x) - (W_{y[i]}^2 + \delta^2W_{x(i)}^2 + 2\delta W_{yx(i)})] \\
\Longrightarrow \; \text{MSE}(\overline{y}_{MM1,\,RSS}) &= \overline{Y}^2[\theta(C_y^2 + \delta^2C_x^2 + 2\delta\rho_{yx}C_yC_x) - (W_{y[i]} + \delta W_{x(i)})^2]
\end{aligned}$$

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Table 2. MSE of different new proposed estimators using RSS

S.No	$W_{y[i]}$	$W_{x(i)}$	$\overline{y}_{MM1,RSS}$	$\overline{y}_{MM2,RSS}$	$\overline{y}_{MM3,RSS}$	$\overline{y}_{MMp1,RSS}$	$\overline{y}_{MMp2,RSS}$	$\overline{y}_{MMp3,RSS}$
1	0.2444163	0.2444163	67392.53	67430.077	67456.8	35664.191	41806.22	43628.698
2	0.1608874	0.2166181	77378.12	77410.624	77450.8	39901.082	47145.4	49299.20
3	0.1541049	0.2043069	81705.615	81736.011	81781.98	41809.469	49521.12	51814.96
4	0.2068323	0.2193567	65317.564	65352.958	65385.18	31711.516	38090.69	40000.02
5	0.2422283	0.2083651	58716.775	58752.27	58784.22	25689.841	31858.93	33717.66
6	0.3034766	0.2187029	37251.122	37291.629	37309.8	10717.008	15474.86	16930.46
7	0.1619454	0.2220363	75859.732	75893.094	75930.91	39343.455	46407.43	48506.54
8	0.1935257	0.3026198	45460.889	45510.248	45504.06	27028.852	30657.69	31720.153
9	0.3016375	0.2751583	18772.637	18824.117	18812.1	4748.8112	7107.726	7842.27
10	0.2379664	0.2312074	53457.176	53496.267	53518.32	24445.787	29862.91	31493.33
11	0.2511795	0.3463367	11307.707	11371.074	11326.36	9519.7583	9812.801	9890.87
12	0.1569224	0.1446283	93256.98	93280.076	93346.13	45251.309	54472.55	57225.897
13	0.1493945	0.2530392	71397.4	71434.847	71461.42	39130.158	45466.27	47335.591
14	0.1993955	0.2052798	70823.032	70855.857	70895.15	34237.787	41199.18	43282.125
15	0.1991578	0.1575615	82064.33	82090.532	82148.04	37973.084	46364.12	48878.01
16	0.2803909	0.3006101	17284.389	17339.799	17316.97	7302.6861	8979.811	9497.61
17	0.1636743	0.1328668	94177.682	94199.739	94268.65	45046.902	54471.4	57287.41
18	0.2149233	0.2483561	55158.454	55199.166	55216.76	27610.762	32829.22	34389.54
19	0.1863996	0.201545	74909.907	74941.528	74984.13	36806.996	44090.2	46265.976
20	0.1821599	0.1904967	78490.993	78520.839	78568.32	38347.973	46025.94	48319.92
21	0.3002727	0.207824	41719.82	41758.204	41782.21	12695.482	17945.88	19547.632
22	0.1460293	0.1686299	90773.205	90798.629	90858.28	45272.48	54045.38	56659.912
23	0.2177058	0.2314364	59128.27	59166.243	59191.37	28566.76	34339.98	36069.976
24	0.2049618	0.2547593	56173.448	56214.667	56230.87	29022.849	34203.78	35747.925
25	0.2059191	0.2264063	37672.87	37725.44	37710.43	23053.42	25917.87	26754.79
26	0.2059191	0.2264063	63696.113	63732.59	63761.84	31288.326	37445.52	39287.143
27	0.2550677	0.2908147	29479.502	29530.989	29518.95	14634.672	17327.72	18140.312
28	0.2036649	0.259029	55329.861	55371.755	55386.1	28902.036	33954	35458.166
29	0.1367108	0.2650215	71575.399	71613.987	71637.42	40300.115	46512.08	48335.707
30	0.2753502	0.2667375	30740.941	30788.939	30786.5	12346.566	15629.13	16630.467
31	0.1272454	0.2317406	81587.13	81619.984	81659.2	43953.303	51345.52	53528.856
32	0.1799369	0.2450833	65633.69	65671.717	65696.71	34533.534	40537.36	42320.549
33	0.1655114	0.2494993	68227.13	68265.005	68290.42	36710.21	42845.4	44661.56
34	0.1347075	0.2078146	85139.85	85169.74	85217.1	44412.05	52337.71	54688.96
35	0.178853	0.1612562	85633.9	85659.76	85718.2	40968.98	49510.62	52064.63
36	0.2062727	0.2981372	42910.11	42959.53	42953.17	24813.35	28321.04	29354.29
37	0.2180059	0.2423964	55975.34	56015.198	56035.14	27526.98	32902.98	34512.58
38	0.1492048	0.1930223	85197.75	85226.38	85277.21	43286.01	51384.31	53794.34
39	0.2272156	0.2679279	45752.24	45797.29	45802.95	23239.79	27474.23	28741.13
40	0.2771293	0.254343	34248.58	34294.25	34298.2	13259.35	17028.34	18177.29
41	0.1859491	0.2434413	64482.1	64520.22	64544.96	33581.98	39526.82	41294.89
42	0.0981292	0.157136	101204.8	101227.06	101295.5	51652.79	61289.91	64153.13
43	0.2381451	0.3049594	30418.67	30471.854	30455.16	17117.834	19597.22	20335.151
44	0.1491502	0.1650475	90861.67	90886.801	90947.26	45064.299	53885.26	56515.349
45	0.2321868	0.299776	34108.63	34160.292	34147.77	19009.764	21848.36	22692.428
46	0.2616354	0.2424136	42998.02	43040.493	43053.25	18149.957	22703.09	24082.143
47	0.246044	0.2691775	39499.1	39545.64	39547.21	18971.906	22763.63	23905.313
48	0.1936005	0.3197516	40114.27	40167.084	40151.4	25439.676	28385.83	29238.293
49	0.2438818	0.1878242	63846.72	63879.056	63919.69	27232.182	34086.86	36151.958
50	0.2794133	0.2945761	19777.91	19831.95	19812.9	8239.82	10213.37	10820.517
51	0.2489161	0.292655	30925.64	30977.06	30965.2	15892.206	18646.11	19473.87
52	0.0774664	0.2203904	93959.08	93987.606	94038.73	50841.118	59402.15	61921.95
53	0.238584	0.3750514	5304.94	5374.03	5313.58	9456.74	8734.38	8491.58
54	0.1644774	0.2711804	62821.24	62862.713	62878.23	35205.168	40635.24	42234.023
55	0.2696805	0.21708	48268.31	48306.712	48330.67	18897.709	24292.27	25927.494
56	0.1303334	0.3290823	56289.47	56339.078	56332.19	36637.069	40821.17	42009.822
57	0.1669058	0.1641195	87514.23	87539.963	87598.75	42585.717	51203.01	53776.581
58	0.2655321	0.2949344	24507.45	24560.554	24544.07	11662.974	13938.64	14630.062
59	0.1619795	0.2367347	72310.26	72345.874	72377.49	38264.327	44875.05	46835.331
60	0.2241839	0.1814654	70571.41	70601.901	70647.6	31636.392	38982.7	41189.659

Table 2. MSE of different new proposed estimators using RSS

S.No	$W_{y[i]}$	$W_{x(i)}$	$\overline{y}_{MM1,RSS}$	$\overline{y}_{MM2,RSS}$	$\overline{y}_{MM3,RSS}$	$\overline{y}_{MMp1,RSS}$	$\overline{y}_{MMp2,RSS}$	$\overline{y}_{MMp3,RSS}$
61	0.2667842	0.2379522	42782.3	42824.264	42838.43	17426.966	22060.64	23466.018
62	0.2777905	0.272527	27972.07	28021.4	28015.31	11113.707	14096.76	15008.694
63	0.2210359	0.3543453	19314.44	19377.053	19334.42	15957.789	16702.17	16896.833
64	0.2472569	0.1929325	61568.14	61601.425	61639.45	26077.023	32707.04	34705.785
65	0.2588797	0.2451435	43007.24	43050.055	43061.88	18499.608	22996.93	24357.998
66	0.2597567	0.2174883	51096.58	51134.519	51159.75	21073.03	26619.78	28297.333
67	0.1269496	0.1310422	100767.7	100788.35	100861.1	49780.23	59621.18	62554.721
68	0.2854493	0.3142526	10484.48	10543.297	10511.11	4406.9194	5330.232	5622.1073
69	0.1253354	0.2342186	81462.11	81495.213	81533.74	44067.286	51423.96	53595.3
70	0.2461632	0.359691	8221.76	8287.83	8235.68	9279.31	9077.73	9000.67
71	0.1684509	0.2309024	72151.01	72186.09	72219.18	37621.35	44297.58	46280.99
72	0.2232215	0.2365153	56161.76	56200.917	56222.8	27004.359	32495.89	34142.798
73	0.209882	0.2269055	62500.79	62537.57	62565.98	30489.41	36559.73	38376.615
74	0.2393712	0.2338076	52294.7	52334.328	52354.92	23902.773	29198.27	30792.5
75	0.1556964	0.1896346	84538.88	84567.368	84618.59	42555.155	50648.48	53059.397
76	0.2025031	0.2209682	66030.01	66065.426	66097.59	32386.464	38785.93	40699.705
77	0.2474396	0.2986217	29422.96	29475.515	29460.55	15612.171	18144.9	18904.499
78	0.227574	0.1653495	73699.08	73727.442	73779.01	32351.664	40157.05	42502.44
79	0.1418001	0.2260361	79602.66	79635.502	79674.75	42198.721	49495.23	51656.334
80	0.2108127	0.2259284	62512.89	62549.565	62578.27	30394.494	36481.96	38304.432
81	0.1730012	0.2762987	59102	59144.95	59156.39	33262.188	38332.54	39825.616
82	0.1647083	0.2172784	76329.21	76362.019	76401.36	39241.094	46401.24	48530.961
83	0.2713578	0.3038717	19343.21	19398.653	19375.73	9223.7915	10964.98	11497.267
84	0.2446024	0.3508385	12073.22	12137.069	12091.02	10656.771	10916.76	10980.063
85	0.277863	0.1731871	58576.13	58607.737	58650.38	21449.735	28334.08	30416.775
86	0.2052837	0.4045614	6749.855	6823.0214	6751.346	14371.373	13236.11	12837.225
87	0.2988562	0.2163892	39468.06	39507.894	39527.9	12107.574	17037.68	18543.318
88	0.2748609	0.2132326	47866.02	47904.024	47929.06	18111.142	23566.18	25221.256
89	0.2082417	0.4050091	5466.226	5539.7997	5467.003	13615.551	12365.62	11933.98
90	0.1660342	0.1721751	86034.39	86061.085	86117.22	42178.226	50595.17	53107.845
91	0.1829896	0.2475286	64184.66	64223.278	64246.64	33787.314	39650.39	41392.033
92	0.2529748	0.2219905	51767.67	51806.01	51830.13	22116.35	27610.71	29270.113
93	0.2435302	0.2454923	47599.79	47641.74	47655.95	21883.407	26659.86	28098.838
94	0.2300127	0.3468285	18792.18	18853.849	18813.8	14633.343	15473.99	15704.873
95	0.2393807	0.2561018	45643.65	45687.28	45696.85	21753.091	26200.13	27537.473
96	0.1948703	0.1894387	75771.41	75801.704	75847.96	36282.906	43804.61	46055.521
97	0.2464348	0.3251931	20627.04	20685.215	20654.78	13016.913	14406.18	14816.314
98	0.1998377	0.3183866	38564.46	38617.481	38601.22	24261.171	27108.04	27934.229
99	0.1755434	0.3537057	35020.97	35079.458	35048.15	26032.465	28056.76	28610.794
100	0.1662525	0.2992661	54537.79	54584.479	54585.64	32612.871	37006.8	38287.146