Chilean Journal of Statistics
Vol. 6, No. 2, September 2015, 19–30

Bayesian Inference
Research Paper

Bayes prediction bound lengths for a repairable system

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(Received: 13 January 2015 · Accepted in final form: 4 September 2015)

Abstract

The objective of the present article is to study about the prediction bound lengths of a considered repairable model under Bayesian approach. The prediction length of bounds and HPD intervals has been obtained under the assumption that the repair failure rate increases monotonically as time parameter increases in the underlying model for censored data.

Keywords: Bayes Prediction Bounds · Lengths · HPD Intervals.

Mathematics Subject Classification: Primary 62A15 · 62F15 · Secondary 65C05.

1. Introduction

The prediction of the future ordered observations shows, how long a sample of units might run until all fail in life testing. In many applications, technical systems or sub - systems have $k - out - of - n$ structure, which has investigated extensively in literature. For such a system, the system consisting of n components or subsystems, of which only $k(\leq n > 0)$ need to be functioning. The $k - out - of - n$ model is commonly used model in reliability theory. In this model, the failure of any component of the system does not influence the components still at work. It finds wide applications in both industrial and military systems. These systems include the multidisplay system in cockpits, multiengine system in an airplane, and the multipurpose system in a hydraulic control system.

The system $(n-1) - out - of - n : G$ is consists with n components and works if and only if $(n-1)$ components among the n, work simultaneously. The system and each of its components can in only one of two states: working or failed. When a component fails, it kept under the repair and the other components stay in the working state with adjusted rates of failure. After repair, a component works as new and its actual lifetime is same as initially. If failed component is repair before another component fails, the $(n-1)$ components recover their initial lifetime. The lifetime and time of repair are independent.

We consider here $1 - out - of - n : G$, system which consists of n components of same kind, with independent and identically distributed life - length. The system is observed under an inspection policy, where inspection is made at the completion of a repair if it starts at beginning of a repair. This leads us to a situation where separate observations

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ISSN: 0718-7912 (print)/ISSN: 0718-7920 (online)

http://www.soche.cl/chjs

on the unit's performance and on repair facility are not feasible. Thus, available records are the number of failures that occurred in the time interval between two repair epochs i.e., the time instant at which a repair completes. Recently, Prakash (2011) have studied some Bayes estimators for $1 - out - of - n : G$, Repairable System.

The objective of the present article is to predict the nature of future behavior of an observation when sufficient information about past and the present behavior of an event or an observation is known or given. We present here a Bayesian statistical analysis to predict the future ordered statistic from the considered repairable model under item failure censored (ordered) data. A good deal of literatures is available on predictive inference of the future failure distribution. Howlader (1985) derived HPD intervals for k^{th} order statistic in the Rayleigh model. Raqab (1997) discussed the prediction problems for the Rayleigh and Normal models. Howlader & Hossain (1995) have considered the Bayesian estimation and prediction of Rayleigh parameter based on Type - II censored data. Sinha (1990), Cramer & Kamps (1998), Raqab & Madi (2002), Sarhan & Tadj (2003), Ali - Mousa & Al - Sagheer (2005), Ahmad et al (2007), Gherda & Boushaba (2011) and Prakash & Singh (2013) are a little few of those who have been extensively studied predictive inference for the future observations.

In present article, we predict the length of bounds for the future ordered statistic under one - sided interval and interval with central coverage for both One - Sample and Two - Samples Bayes prediction scenario. The highest posterior density intervals are also obtained, and a simulation study has been done for illustration of proposed methods. Application on real data of the proposed methods have also discussed.

2. Description of the model under study

The considered repairable system is based on following assumptions:

The system consists with n units and having a repair facility. Initially one unit starts operating and remaining $(n-1)$ units are kept as inactive standbys. As soon as a unit fails, it goes for repair and a standby unit is put on the operation. The repair policy is based on *First Come First Serves* (FCFS), it is always open, and the repairs are perfect with negligible switch over time.

The failure time distribution of online units and the repair time distribution of units under repair are assume general, independent of each other and both are increasing failure rate (IFR) distributions. The number of non - operative units defines the state of the system, in the system at the time $t(> 0)$. Further, the state n is called the down state of the system.

The system is observed under an inspection policy where inspection is made at the completion of a repair, if it starts at the beginning of a repair. This leads us to a situation where separate observations on the unit's performance and on repair facility are not feasible. Thus, available records are the number of failures that occurred in the time interval between two repair epochs i.e., the time instant at which a repair completes.

Let, $X(t) = i; i = 0, 1, ..., n$ be the number of non-operative units at time t. The system is putting on a test and is observed continuously until n repairs are over. Let the system be in state $i(= 1, 2, ..., n-1)$ initially whenever a first repair starts and the states of the process are recorded at the completion of each repair. The failure rate $\rho_{ik}(t)$ of the system at the state k if it was in the state i is given as

$$\rho_{ik}(t) = \frac{n_{ik}}{u+1} t^v; \; u+1 > n_{ik}, \; v > 0. \tag{1}$$

Here, n_{ik} be sample size obtained from the system when system passes through the state

i to state k, and u be the number of units failed when the system transits from the state i to the state k. Here, the failure rate decreases with u increases. Re - parameterization of failure rate is given as

$$\rho(t) = \theta t^{v} \; ; \; \theta = \frac{n_{ik}}{u+1}. \tag{2}$$

In life testing, the observations usually occurred in ordered manner such a way that weakest items fail first and then second one and so on. Let us suppose that $n(\geq 3)$ items are put to test under the model without replacement and test terminates as soon as first $r^{th}(\leq n)$ item fails.

Let $x_{(1)}, x_{(2)}, ..., x_{(r)}(= \underline{x})$ be the first r observed ordered item from the model having the likelihood function

$$L\left(\underline{x}|\theta\right) \propto \left(\prod_{j=1}^{r} f\left(x_{(j)};\theta\right)\right) \cdot \left(\frac{f\left(x_{(r)};\theta\right)}{\rho\left(x_{(r)}\right)}\right)^{n-r}$$

$$\Rightarrow L\left(\underline{x}|\theta\right) \propto \theta^{r} \, A_r(\underline{x}) \, e^{-\theta T_r(\underline{x})} \; ; \tag{3}$$

where $A_r(\underline{x}) = \prod_{j=1}^{r} x_{(j)}^{v}$ and $T_r(\underline{x}) = \frac{1}{v+1}\left(\sum_{j=1}^{r} x_{(j)}^{v+1} + (n-r)x_{(r)}^{v+1}\right)$ is a sufficient statistic for parameter θ.

From a Bayesian viewpoint, there is clearly no way in which one can say that one prior is better than other. We select to restrict attention to a given flexible family of priors, and choose one from that family, which seems to match best with our personal beliefs. One of best choice for selecting the prior distribution is conjugate prior. We consider here Gamma distribution as the conjugate prior for the parameter θ with probability density function

$$\pi(\theta) \propto \theta^{\alpha-1} \, exp\left(-\beta\theta\right); \; \alpha > 0, \beta > 0, \theta > 0. \tag{4}$$

3. Bayes prediction bound lengths under One - Sample plan

Let $x_{(1)}, x_{(2)}, ..., x_{(r)}$ be the first r components of the observed ordered items from a sample of size n. If $Y = \left(y_{(1)}, y_{(2)}, ..., y_{(m)}\right)$ be the second independent ordered random sample of the future observations from the same model. Then Bayes predicative density of future observation Y is denoted by $h\left(y|\underline{x}\right)$ and obtained by simplifying

$$h\left(y|\underline{x}\right) \propto \int_{\theta} f\left(y;\theta\right) \pi^{*}\left(\theta|\underline{x}\right) d\theta \; ; \tag{5}$$

where $\pi^{*}\left(\theta|\underline{x}\right)$ be the posterior density of the parameter θ obtained under prior given in equation (4).

Solving (5), we have

$$h\left(y|\underline{x}\right) \propto \left(T_r^{*}(\underline{x})\right)^{r+\alpha} y^{v} \left(T_r^{*}(\underline{x}) + \frac{y^{v+1}}{v+1}\right)^{-(r+\alpha+1)} \; ; \; T_r^{*}(\underline{x}) = T_r(\underline{x}) + \beta \tag{6}$$

The Bayes predictive density function $h\left(y|\underline{x}\right)$ expresses the plausibility of Y given data and prior information regarding the parameter.

The Bayes predictive bounds with coverage $(1-\tau)$ is defined for future observation Y as

$$Pr\left(l_1 \leq Y \leq l_2\right) = 1 - \tau. \tag{7}$$

Here l_1 and l_2 are the lower and upper Bayes prediction bounds for random variable Y, and $(1-\tau)$ is called the confidence prediction coefficient.

The One - sided $100(1-\tau)\%$ Bayes prediction bounds are obtained by solving following equality

$$Pr\left(Y \leq l_1\right) = \frac{\tau}{2} = Pr\left(Y \geq l_2\right). \tag{8}$$

Solving (8), the lower and upper prediction bounds for Y are obtain as

$$l_1 = \left\{(v+1)\left(T_r^*\left(\underline{x}\right)\right)\tau^*\right\}^{\eta}$$

and

$$l_2 = \left((v+1)\left(T_r^*\left(\underline{x}\right)\right)\tau^{**}\right)^{\eta} \ ;$$

where $\tau* = (\tau_1)^{-\lambda} - 1$, $\tau** = (\tau_2)^{-\lambda} - 1$, $\eta = (v+1)^{-1}$, $\lambda = (r+\alpha)^{-1}$, $\tau_1 = \left(1 - \frac{\tau}{2}\right)$ and $\tau_2 = \left(\frac{\tau}{2}\right)$.

Hence, the Bayes prediction length of bounds is

$$I = l_2 - l_1. \tag{9}$$

Now, the central coverage $100\,\tau\%$ Bayes prediction bounds for future observation Y are obtained similarly by solving following equality

$$Pr\left(Y \leq l_{1C}\right) = \frac{1-\tau}{2} = Pr\left(Y \geq l_{2C}\right). \tag{10}$$

Solving (10), the lower and upper Bayes prediction bounds and length of Bayes prediction bounds for future random observation Y are given as

$$l_{1C} = \left((v+1)\left(T_r^*\left(\underline{x}\right)\right)\omega^*\right)^{\eta} \ ,$$
$$l_{2C} = \left((v+1)\left(T_r^*\left(\underline{x}\right)\right)\omega^{**}\right)^{\eta}$$

and

$$I_C = l_{2C} - l_{1C} \ ; \tag{11}$$

where $\omega^* = \left(\left(\frac{1+\tau}{2}\right)^{-\lambda} - 1\right)$ and $\omega^{**} = \left(\left(\frac{1-\tau}{2}\right)^{-\lambda} - 1\right)$.

4. Bayes prediction bound lengths under Two - Sample plan

We have first r observed ordered failure items $x_{(1)}, x_{(2)}, ..., x_{(r)}$ from the considered model. If $y_{(1)}, y_{(2)}, ..., y_{(m)}$ is the second (unobserved) censored data of size m drawn independently from same model, then the first sample is referred to as the informative sample, while the second one is referred to as the future sample. Based on an informative sample, our aim is to predict the j^{th} order statistic from future sample.

Based on the Bayes predictive density (6) of the future observation Y, the cumulative density function is defined and obtained as

$$G(y|\underline{x}) = Pr(Y \leq y) = \int_0^y h(y|\underline{x})\, dy$$

$$\Rightarrow G(y|\underline{x}) = 1 - \left(1 + \frac{y^{v+1}}{(v+1)T_r^*(\underline{x})}\right)^{-\lambda^{-1}}. \tag{12}$$

Now, if Y_j denote the j^{th} order statistic in future sample of size $m\,(1 \leq j \leq m)$ then from m future observations, the probability density function of the j^{th} ordered future observation is obtain from

$$\phi(Y_j) = j\binom{m}{j}\left(G(y|\underline{x})\right)^{j-1}\left(1 - G(y|\underline{x})\right)^{m-j} h(y|\underline{x}). \tag{13}$$

The prediction limits for Y_j, the j^{th} smallest item of a set of m future ordered observations follows the probability density function (13). The lower and upper Bayes prediction one - sided $100(1-\tau)\%$ bounds for j^{th} item is obtained from following equality

$$Pr(Y \leq l_{1j}) = \frac{\tau}{2} = Pr(Y \geq l_{2j}). \tag{14}$$

The lower and upper Bayes prediction one - sided $100(1-\tau)\%$ bounds for j^{th} item are the solution of the following equalities obtained from (14)

$$j\binom{m}{j}\int_0^{l_1} z^{j-1}(1-z)^{m-j}\, dz = \frac{\tau}{2} \tag{15}$$

and

$$j\binom{m}{j}\int_0^{l_2} z^{j-1}(1-z)^{m-j}\, dz = 1 - \frac{\tau}{2} \tag{16}$$

where $l_i = 1 - \left(1 + \frac{l_{ij}^{v+1}}{(v+1)T_r^*(\underline{x})}\right)^{-\lambda^{-1}}$ and $i = 1, 2$.

For first future observation (e.g., $j = 1$) the Bayes prediction lower and upper bounds are obtained as

$$l_{11} = \{(v+1)T_r^*(\underline{x})\xi^*\}^{\eta}$$

and

$$l_{21} = \{(v+1)T_r^*(\underline{x})\xi^{**}\}^{\eta},$$

where $\xi^* = (\tau^* + 1)^{1/m} - 1$ and $\xi^{**} = (\tau^{**} + 1)^{1/m} - 1$.

Therefore, the Bayes prediction length of bounds for smallest future observation is

$$I = l_{21} - l_{11}. \tag{17}$$

Similarly, the Bayes prediction bounds for last future observation (e.g., $j = m$) are obtained as

$$l_{1m} = \left\{(v+1)T_r^*(\underline{x})\xi^+\right\}^\eta$$

and

$$l_{2m} = \left\{(v+1)T_r^*(\underline{x})\xi^{++}\right\}^\eta,$$

where $\xi^+ = \left(1 - \tau_2^{1/m}\right)^{-\lambda} - 1$ and $\xi^{++} = \left(1 - \tau_1^{1/m}\right)^{-\lambda} - 1$.

The Bayes prediction length of bounds for largest future observation is

$$I = l_{2m} - l_{1m}. \tag{18}$$

On similar lines the central coverage Bayes prediction bounds under the two - sample plan are given for smallest future observations as

$$l_{11C} = \left\{(v+1)T_r^*(\underline{x})\omega^+\right\}^\eta$$

and

$$l_{21C} = \left\{(v+1)T_r^*(\underline{x})\omega^{++}\right\}^\eta,$$

where $\omega^+ = (\omega^* + 1)^{1/m} - 1$ and $\omega^{++} = (\omega^{**} + 1)^{1/m} - 1$.

The central coverage Bayes prediction bounds for largest future observations are obtained on similar line as

$$l_{1mC} = \left\{(v+1)T_r^*(\underline{x})\omega'\right\}^\eta$$

and

$$l_{2mC} = \left\{(v+1)T_r^*(\underline{x})\omega''\right\}^\eta \ ;$$

where $\omega' = \left(1 - \left(\frac{1-\tau}{2}\right)^{1/m}\right)^{-\lambda} - 1$ and $\omega'' = \left(1 - \left(\frac{1+\tau}{2}\right)^{1/m}\right)^{-\lambda} - 1$.

The central coverage prediction length corresponding to first and last future observation under two - sample plan are given respectively as

$$I_C = l_{21C} - l_{11C} \tag{19}$$

and

$$I_C = l_{2mC} - l_{1mC} \tag{20}$$

5. Highest posterior density intervals

In this section, our objective is to study about the highest posterior density (HPD) interval for unknown parameter θ under considered repairable model. Since, the posterior density $\pi^*(\theta|\underline{x})$ corresponding to the parameter θ is unimodel.

Thus, $100(1-\tau)\%$ HPD interval $[H_1, H_2]$ for parameter θ must satisfy the following equations simultaneously.

$$\int_{H_1}^{H_2} \pi^* (\theta|\underline{x})\, d\theta = 1-\tau \tag{21}$$

and

$$\pi^* (H_1|\underline{x}) = \pi^* (H_2|\underline{x}). \tag{22}$$

Now, expressions (21) & (22) are rewritten as

$$\int_{H_1(T_r^*(\underline{X}))}^{H_2(T_r^*(\underline{X}))} \frac{1}{\Gamma(\alpha+r)} e^{-z} z^{\alpha+r-1} dz = 1-\tau$$

$$\Rightarrow \frac{1}{\Gamma(\alpha+r)} \left[\gamma\left\{(\alpha+r),(H_2T_r^*(\underline{X}))\right\} - \gamma\left\{(\alpha+r),(H_1T_r^*(\underline{X}))\right\}\right] = 1-\tau \tag{23}$$

and

$$\left(\frac{H_2}{H_1}\right)^{\alpha+r-1} = e^{-(H_1-H_2)T_r^*(\underline{X})}. \tag{24}$$

Solve simultaneously the equations (23) & (24) to obtain the highest posterior density limits H_1 and H_1.

6. Numerical analysis based on Simulated data

We illustrate the procedure by presenting a complete analysis under a simulated data set. The random samples are generated as follows:

Case 1: One - Sample Plan

(1) Generate θ trough prior density $\pi(\theta)$ for a selected set of prior parametric values $(\alpha, \beta) = (0.25, 0.50), (1.00, 1.00), (2.00, 1.40)$ and $(4.00, 2.00)$. The values of (α, β) are chosen here so as to keep the prior variance unity.

(2) Using θ obtained from step (1) with $v = 1$; a set of $10,000$ random samples of size $n = 15$ has been drawn from the model.

(3) For the selected values of censored sample size $r = 04, 06, 08, 12, 15$, and level of significance $\tau = 99\%, 95\%, 90\%$; the Bayes prediction lengths of bounds are obtain for both cases and presented them in Table (1).

(4) It is observed from the table that, the lengths of Bayes prediction bounds under One - Sample plan for both cases tend to be wider as the censored sample size r increases when other parametric values are fixed.

(5) The length of bounds expended also, when combination of prior parameter (α, β) increases.

(6) It is also seen that the length of bounds tends to be closer when the level of significance τ decreases for both cases when other parametric values are fixed.

(7) It is observed also that the central coverage Bayes prediction length of bounds tends to be closer as compare to one–sided Bayes prediction length of bounds. This shows that the lengths of bounds under the central coverage criterion are robust.

Case 2: Two - Sample Plan

The random samples are generated under Two - Sample plan as follows:

(1) A set of $10,000$ random samples of size $n = 15$ has been drawn from the model for the similar set of parametric values as consider earlier in section 4.
(2) The Bayes prediction lengths of bounds are obtain for smallest and largest future observation for both cases and presented in Tables (2 - 3).
(3) The properties of the Bayes prediction length of bounds under Two - Sample plan for one - sided and central coverage prediction lengths have been seen similar to the One - Sample plan respectively.
(4) It is also noted that the length of intervals tend to wider as compare to the smallest and largest observations. This is a natural, since the prediction of the future order statistic that is far away from the last observed value and has less accuracy than that of other future order statistics.
(5) It is also observed that the central coverage Bayes prediction length of interval tends to be closer as compare to one sided prediction length of interval. This shows that the lengths of intervals under the central coverage criterion are robust.

Case 3: HPD Intervals

(1) The HPD limits are obtain for a set of $10,000$ random samples of size $n = 15$ which has been drawn from the considered model for the similar set of parametric values as consider earlier and presented in Table (4).
(2) It is observed from the table that the HPD limit becomes narrower as the prior parameter (α, β) increases.
(3) Similar trend also has been seen when censored sample size r increases when other parametric values are considered to be fixed.
(4) Further, the HPD length of interval becomes wider as the coverage probability τ increases.
(5) It is also observed that the interval is least.

Remark. In case when the censored sample size $r(= 15)$ the censoring criterion is reduces to complete sample size criterion and hence all the result are valid for complete sample case.

7. Real Life Example

A model is a useful representation that captures the essence of a real system and behaves sufficiently like it in such a way that conclusions can be drawn from the model's behavior to aid in making prudent decisions about the real system. Situations in which the $k-out-of-n : G(F)$ system serves as a useful model are

(1) A piece of stranded wire with n strands in which at least k strands are necessary to pass the required current behaves as a $k-out-of-n : G$ system. The same concept generalizes to applications involving *supply - type* components with identical fixed ratings for their *capacity*, *flow*, *throughput* and *strength*, such that system success is achieved when a minimum supply is met, or when a certain threshold is exceeded
(2) A three - engine airplane which can stay in the air if and only if at least two of its three engines are functioning is a $2 - out - of - 3 : G$.
(3) A space vehicle requiring three out of its four main engines to operate in order to achieve orbit is a $3 - out - of - 4 : G$ system.

The above two example are the system that tolerates the failure of one (and only one) component, since such a failure reduces the system to a series system, which is still a working system.

(4) Reactor protection systems, sensor systems, alarm generation systems and other decision mechanisms usually employ a $k - out - of - n : G$ system.
(5) A bridge with n main supports that can survive an earthquake if and only if at least k supports remain intact is approximately modeled as a $k-out-of-n : G$ system. Here, the modeling is *qualitative* rather than quantitative, since a bridge is usually not structurally symmetric with respect to its supports, while a $k - out - of - n$ system is structurally symmetric with respect to its components.

A practical application of this model is presented in this paper by using example considered by Tiana et al. (2008). The numerical finding are presented in the Table (5).

The finding are presented here for sample size $n(= 6)$ and considered prior parameters $(\alpha, \beta) = (1.00, 1.00)$. Other parametric values are chosen similar as discussed above as required. All the properties discussed above under both sample plans for one-sided and central coverage are seen to be similar.

Table 1. Bayes Prediction Length of Intervals under One-Sample Technique

$n = 15$		**One - Sided**			**Central Coverage**		
r	$(\alpha, \beta) \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
04	0.25,0.50	0.5924	0.4585	0.3870	0.5747	0.4448	0.3715
	1.00,1.00	0.8818	0.6774	0.5739	0.8554	0.6571	0.5509
	2.00,1.40	1.0927	0.8420	0.7119	1.0600	0.8167	0.6834
	4.00,2.00	1.4268	1.0996	0.9300	1.3839	1.0666	0.8928
06	0.25,0.50	0.6229	0.4816	0.4072	0.6042	0.4671	0.3909
	1.00,1.00	0.8998	0.6929	0.5874	0.8727	0.6721	0.5638
	2.00,1.40	1.1101	0.8563	0.7237	1.0768	0.8306	0.6947
	4.00,2.00	1.4397	1.1108	0.9389	1.3965	1.0774	0.9013
08	0.25,0.50	0.6357	0.4895	0.4140	0.6166	0.4748	0.3975
	1.00,1.00	0.9091	0.7003	0.5922	0.8818	0.6793	0.5685
	2.00,1.40	1.1158	0.8601	0.7279	1.0823	0.8344	0.6988
	4.00,2.00	1.4451	1.1138	0.9422	1.4017	1.0803	0.9045
12	0.25,0.50	0.8295	0.6287	0.5282	0.7963	0.6098	0.4965
	1.00,1.00	1.2148	0.9316	0.7848	1.1662	0.9036	0.7377
	2.00,1.40	1.5169	1.1601	0.9790	1.4562	1.1253	0.9203
	4.00,2.00	1.9809	1.5169	1.2786	1.9016	1.4713	1.2018
15	0.25,0.50	0.8651	0.6619	0.5594	0.8305	0.6421	0.5258
	1.00,1.00	1.2556	0.9578	0.8062	1.2054	0.9290	0.7578
	2.00,1.40	1.5451	1.1781	0.9931	1.4833	1.1427	0.9335
	4.00,2.00	2.0088	1.5301	1.2882	1.9285	1.4842	1.2109

Table 2. Bayes Prediction Length of Intervals under Two-Sample Technique (Smallest future Observation)

$n = 15$		**One - Sided**			**Central Coverage**		
r	$(\alpha, \beta) \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
04	0.25,0.50	0.5066	0.3751	0.3500	0.6050	0.4655	0.4426
	1.00,1.00	0.7503	0.5522	0.5138	0.8983	0.6910	0.6523
	2.00,1.40	0.9315	0.6870	0.6389	1.1144	0.8585	0.8132
	4.00,2.00	1.2161	0.8983	0.8332	1.4582	1.1205	1.0603
06	0.25,0.50	0.5306	0.3915	0.3649	0.6384	0.4916	0.4634
	1.00,1.00	0.7685	0.5678	0.5262	0.9202	0.7078	0.6688
	2.00,1.40	0.9457	0.6994	0.6484	1.1337	0.8730	0.8244
	4.00,2.00	1.2279	0.9078	0.8409	1.4709	1.1321	1.0701
08	0.25,0.50	0.5402	0.3995	0.3710	0.6489	0.4997	0.4724
	1.00,1.00	0.7735	0.5718	0.5308	0.9272	0.7140	0.6750
	2.00,1.40	0.9511	0.7031	0.6514	1.1398	0.8773	0.8294
	4.00,2.00	1.2320	0.9101	0.8432	1.4756	1.1358	1.0734
12	0.25,0.50	0.7227	0.5339	0.4958	0.8268	0.6426	0.6111
	1.00,1.00	1.0781	0.7923	0.7320	1.2548	0.9502	0.9046
	2.00,1.40	1.3458	0.9842	0.9196	1.5594	1.1873	1.1248
	4.00,2.00	1.7641	1.2854	1.2012	2.0321	1.5462	1.4685
15	0.25,0.50	0.7740	0.5585	0.5226	0.8861	0.6774	0.6438
	1.00,1.00	1.1124	0.8140	0.7557	1.2816	0.9756	0.9269
	2.00,1.40	1.3737	1.0032	0.9332	1.5820	1.2028	1.1391
	4.00,2.00	1.7820	1.3019	1.2098	2.0540	1.5611	1.4801

Table 3. Bayes Prediction Length of Intervals under Two-Sample Technique (Largest future Observation)

$n = 15$		**One - Sided**			**Central Coverage**		
r	$(\alpha, \beta) \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
04	0.25,0.50	0.6102	0.4518	0.4216	0.7287	0.5607	0.5331
	1.00,1.00	0.9037	0.6651	0.6189	1.0820	0.8323	0.7857
	2.00,1.40	1.1220	0.8275	0.7696	1.3423	1.0341	0.9795
	4.00,2.00	1.4648	1.0820	1.0036	1.7564	1.3496	1.2771
06	0.25,0.50	0.6391	0.4716	0.4395	0.7690	0.5921	0.5582
	1.00,1.00	0.9257	0.6839	0.6338	1.1084	0.8525	0.8056
	2.00,1.40	1.1391	0.8424	0.7810	1.3655	1.0515	0.9930
	4.00,2.00	1.4790	1.0934	1.0129	1.7717	1.3636	1.2889
08	0.25,0.50	0.6507	0.4812	0.4469	0.7816	0.6019	0.5690
	1.00,1.00	0.9317	0.6887	0.6394	1.1168	0.8600	0.8130
	2.00,1.40	1.1456	0.8469	0.7846	1.3729	1.0567	0.9990
	4.00,2.00	1.4839	1.0962	1.0156	1.7774	1.3681	1.2929
12	0.25,0.50	0.8705	0.6431	0.5972	0.9959	0.7740	0.7361
	1.00,1.00	1.2986	0.9543	0.8817	1.5114	1.1445	1.0896
	2.00,1.40	1.6210	1.1855	1.1077	1.8783	1.4301	1.3548
	4.00,2.00	2.1249	1.5483	1.4469	2.4477	1.8624	1.7688
15	0.25,0.50	0.9323	0.6727	0.6295	1.0673	0.8159	0.7755
	1.00,1.00	1.3399	0.9805	0.9102	1.5437	1.1751	1.1165
	2.00,1.40	1.6546	1.2084	1.1240	1.9055	1.4488	1.3721
	4.00,2.00	2.1464	1.5681	1.4572	2.4741	1.8804	1.7828

Table 4. Highest Posterior Density (HPD) Limits

$n = 15$		H_1			H_2		
r	$(\alpha, \beta) \downarrow \tau \rightarrow$	99%	95%	90%	99%	95%	90%
04	0.25,0.50	1.2063	1.1903	1.1746	2.3443	2.2688	2.1956
	1.00,1.00	1.1940	1.1781	1.1624	2.2426	2.1704	2.1005
	2.00,1.40	1.1831	1.1674	1.1518	2.1613	2.0917	2.0243
	4.00,2.00	1.1341	1.1191	1.1042	2.0287	1.9635	1.9003
06	0.25,0.50	1.0901	1.0756	1.0614	2.2129	2.1416	2.0727
	1.00,1.00	1.0847	1.0703	1.0561	2.1327	2.0640	1.9974
	2.00,1.40	1.0291	1.0154	1.0020	2.0019	1.9374	1.8750
	4.00,2.00	1.0936	1.0793	1.0649	1.9213	1.7517	1.6636
08	0.25,0.50	1.0582	1.0443	1.0304	1.8561	1.7016	1.6230
	1.00,1.00	0.9935	0.9802	0.9672	1.7795	1.6313	1.5559
	2.00,1.40	1.0535	1.0395	1.0257	1.8804	1.7145	1.6282
	4.00,2.00	1.0426	1.0289	1.0151	1.8459	1.6854	1.6072
12	0.25,0.50	0.9904	0.9773	0.9643	1.7417	1.5967	1.5229
	1.00,1.00	0.9640	0.9513	0.9387	1.7816	1.6568	1.5932
	2.00,1.40	0.9464	0.9339	0.9215	1.7613	1.6420	1.5823
	4.00,2.00	0.9328	0.9205	0.9083	1.7450	1.6284	1.5701
15	0.25,0.50	1.2063	1.1903	1.1746	2.3443	2.2688	2.1956
	1.00,1.00	1.1940	1.1781	1.1624	2.2426	2.1704	2.1005
	2.00,1.40	1.1831	1.1674	1.1518	2.1613	2.0917	2.0243
	4.00,2.00	1.1341	1.1191	1.1042	2.0287	1.9635	1.9003

Table 5. Bayes Prediction Bound Lengths

	One - Sided			Central Coverage		
	99%	95%	90%	99%	95%	90%
One Sample Plan	0.7596	0.682	0.5105	0.6912	0.5223	0.596
Two Sample Plan						
Smallest Observation	0.497	0.4904	0.4857	0.4771	0.4914	0.4884
Largest Observation	0.873	0.864	0.8566	0.8452	0.8654	0.8612
HPD Interval	H_1			H_2		
	0.8059	0.7988	0.7929	0.7965	0.7949	0.7839

8. Conclusion

In the present article we study about the prediction bound lengths of a repairable model under Bayesian approach. The Bayes prediction length of bounds and HPD intervals has been obtained under assumption that the repair failure rate increases monotonically as time parameter increases. An item-failure censored data used for the study. Both One-Sample and Two-Sample Bayes prediction criterion have used. Also the Bayes prediction lengths of bounds are obtained under one sided and central coverage criterion. A simulation study has been carried out for analyzing the procedures. Examples based on Real data set are discussed herewith. The lengths of bounds under the central coverage criterion are seen here robust. It is also observed that the central coverage Bayes prediction length of interval tends to be closer as compare to one - sided prediction length of interval.

Acknowledgments

The author would like to thank the editor and anonymous referees for their suggestions and valuable comments that led to a significant improvement of this manuscript.

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