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On a Double Reparametrization for Accelerated Lifetime Testing

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Abstract

Accelerated life tests are efficient reliability industrial experiments. In this paper we describe the effect of several reparametrizations on the accuracy of the interval estimation of the parameter of interest when accelerated life tests are considered in presence of small and moderate sized samples. We propose a double reparametrization which leads to accuracy while allows orthogonality between the parameters. The idea is to consider a logarithmic reparametrization on orthogonal parameters in order to have independent maximum likelihood estimates with good asymptotic normal approximation. A simulation study reveals that the coverage probability of the confidence intervals based on the double parametrization are close to the nominal coverage probability even when only small or moderate-size data sets are available and the censoring pattern is heavy. It is the paper endeavour to bring the double reparametrization approach to the attention of reliability analysis practitioners. The methodology is illustrated on a real dataset on an accelerated life test at pressurized containers of Kevlan/Epoxy 49.

Keywords: Accelerated life tests · Accuracy of interval estimation · Asymptotic Normality · Exponential distribution · Likelihood · Parametrization.

Mathematics Subject Classification: Primary 62N05.

1. Introduction

Accelerated life tests (ALT) are usually performed for testing industrial products which may have an excessive duration, thus generating a high time and cost to be tested. In an ALT the items are tested at higher stress covariate levels than the usual working conditions, and then information on the item performance under the usual working conditions, which is the main ALT objective, can be obtained.

There is a large literature on ALT and interested readers can refer to Mann, Schaffer and Singpurwalla (1974), Nelson (1990), Meeker and Escobar (1998), Lawless (2003) which are excellent sources for ALT. Nelson (2005a, 2005b) provides a brief background on

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accelerated testing and test plans and surveys the related literature point out more than 150 related references. Interested readers can refer to Louzada-Neto (2010) which provides a brief introduction to ALT.

Let k be the number of groups of n_i items each under a constant and fixed stress covariate levels, X_i (hereafter stress level), for $i = 1, ..., k$. The first level $i = 1$ is regarded as the usual level and it shall be conveniently named u. The ALT ends after a certain pre-fixed number $r_i < n_i$ of failures, $t_{i1}, t_{i2}, ..., t_{ir_i}$, at each stress level, characterizing a type II censuring scheme (Louzada-Neto, 2010). Others stress schemes, such as step and progressive, are also common in practice but will not be considered here. They however can be found in Nelson (1990) and Meeker and Escobar (1998).

The ALT models take into account two components. The first is probabilistic one. A lifetime distribution, such as the exponential, Weibull, log-normal, log-logistic, gamma, among others. The second component is a stress-response relationship (SRR), which relates the mean lifetime (or a function of this parameter) with the stress levels. Following Mann, Schaffer and Sigpurwalla (1974), the most common SRRs are the power law, Eyring and Arrhenius models. In this paper we assume an exponential distribution as the lifetime model and a general log-linear SRR. The main purpose here is to estimate the mean lifetime under the usual working condition, denoted by θ_u, which is regarded as the parameter of interesting. In this context, maximum likelihood estimation is a usual approach, which shall be consider here.

At this point, a problem arises on the accuracy of interval estimation of θ_u when the asymptotic normality of the maximum likelihood estimates (MLEs) is considered. A natural manner of approaching this problem is searching for an one-to-one transformation of θ_u which gives approximate normality for the likelihood function (Sprott, 1973; Sprott, 1980). In ALT, despite sparse literature, considering the Eyring model, Louzada-Neto and Pardo-Fernandéz (2001) study different parametrizations in order to verify the effect of reparametrization on the accuracy of inferences for the model parameters. Although, they do not examine the coverage probabilities (CP) of the asymptotic confidence intervals obtained under the different parametrizations. Such examination is crucial for verifying the adequacy of asymptotic theory when small or moderate-sized samples are considered, which is usual for ALT.

In this paper we generalize the ideas of Louzada-Neto and Pardo-Fernandéz (2001) by studying the effect of several reparametrizations on the accuracy of the interval parameter estimation of θ_u when a general log-linear model for ALT is considered, which has the power law, Eyring and Arrhenius models as particular cases. We propose a double reparametrization which lead to interval estimation accuracy while allow orthogonality between the parameters. The idea is to consider a logarithmic reparametrization on orthogonal parameters (Cox and Reid, 1987) in order to have independent MLEs with good asymptotical normal approximation, even if the sample size is small or moderate. Moreover, a simulation study is performed in order to examine the CPs of the asymptotic confidence intervals obtained unde different parametrizations.

The paper is organized as following. In Section 2 we present the ALT modeling. In Section 3 we provide the basic concepts involved on using asymptotic results for interval estimation and orthogonal parameters. Some candidate reparametrizations are presented in Section 4, where a double parametrization is proposed. Section 5 contains the results of a simulation study which examines the coverage probabilities of the confidence intervals for the parameters obtained by considering all the studied parametrizations. In Section 6 we consider an example on an ALT on Kevelan/Epoxy 49 pressurized vasels extract from Barlow, Toland and Freeman (1986). Final remarks in Section 7 conclude the paper.

2. The ALT Model Formulation

Let T be the lifetime random variable with exponential density given by

$$f(t, \lambda_i) = \lambda_i \exp\{-\lambda_i t\}, \tag{1}$$

where $\lambda_i > 0$ is an unknown parameter representing the constant failure rate for $i = 1, ..., k$ (number of stress levels).

The mean lifetime is given by

$$\theta_i = 1/\lambda_i. \tag{2}$$

The likelihood function for λ_i, under the i-th stress level X_i is given by

$$L_i(\lambda_i) = \left(\prod_{j=1}^{r_i} f(t_{ij}, \lambda_i)\right)(S(t_{ir_i}, \lambda_i))^{n_i - r_i} = \lambda_i^{r_i} \exp\{-\lambda_i A_i\}, \tag{3}$$

where $S(t_{ir_i}, \lambda_i)$ is the survival function at t_{ir_i} and $A_i = \sum_{j=1}^{r_i} t_{ij} + (n_i - r_i)t_{ir_i}$ denotes the total time on test for the i-th stress level. As noticed by a Referee, censoring influences in (3) on a base imposed by $S(t_{ir_i}, \lambda_i)$ weigh by the amount of censoring, $(n_i - r_i)$. As we shall observe from the simulation results, the heavy censoring will have a remarkable impact on the coverage probabilities of the asymptotic confidence intervals for the parameters of the model.

Considering data under the k random stress levels, the likelihood function for the parameter vector $\lambda = (\lambda_1, \lambda_2, ..., \lambda_k)$ is

$$L(\lambda) = \prod_{i=1}^{k} \lambda_i^{r_i} \exp\{-\lambda_i A_i\}. \tag{4}$$

We define the general log-linear SRR as

$$\lambda_i = \exp(-Z_i - \beta_0 - \beta_1 X_i), \tag{5}$$

where X is the covariate $Z = g(X)$ and β_0 and $\beta_1$are unknown parameters such that -$\infty < \beta_0$, $\beta_1 < \infty$.

The SRR (5) has several usual SRR models as particular cases and it is a directly generalization of Louzada-Neto and Pardo-Fernandéz (2001). The Arrhenius model is obtained if $Z_i = 0$, $Xi = 1/Vi$, β_0=$-\alpha_1$ and $\beta_1 = \alpha_2$, where Vi denotes a level of the temperature variable. If $Z_i = 0$, $Xi = -log(Vi)$, $\beta_0 = log(\alpha)$ and $\beta_1 = \alpha_2$, where Vi denotes a level of the voltage variable we obtain the power model. Following Louzada-Neto and Pardo-Fernandéz (2001), the Eyring model is obtained if $Z_i = -\log V_i$, $Xi = 1/Vi$, β_0=$-\alpha_1$ and $\beta_1 = \alpha_2$, where Vi denotes a level of the temperature variable. Interested readers can refer to Meeker and Escobar (1998) for more information about the physical models considered here.

The MLEs of λ can be obtained by direct maximization of (4), or by solving the system of nonlinear equations, $\partial \log L/\partial\lambda = 0$. The derivatives are not given explicitly, since major simplification is only possible for $k = 2$.

3. On the Requirements for Inferences

Following (Cox and Hinkley, 1974) and (Cox and Reid, 1987), the two important desired requirements for inferences about the parameters is accuracy on the asymptotic normal approximation of the MLEs and the present of orthogonality between them.

Let us consider the bidimensional paramenter case, which is enough for the development of the ideas here. Consider a bidimensional paramenter vector $\mu^T = (\mu_1, \mu_2)$ which is our case. The concepts which will be described here however remains valid for the multivariate case. Moreover, let $L(\mu)$ be the correspondent likelihood function and $l(\mu) = \log L(\mu)$ and let $\widehat{\mu}$ be the MLE of μ. For inferences about μ we can use the asymptotic normality of the MLEs (Cox and Hinkley, 1974),

$$\widehat{\mu}^T = (\widehat{\mu_1}, \widehat{\mu_2}) \xrightarrow{d} N_2\left((\mu_1, \mu_2), I^{-1}(\mu_1, \mu_2)\right), \tag{6}$$

where the limiting argument is the sample size and $I(\mu_1, \mu_2)$ is the Fisher information matrix (FIM) given by the symetric matrix

$$I(\mu_1, \mu_2) = \begin{pmatrix} I_{11}(\mu_1, \mu_2) & I_{12}(\mu_1, \mu_2) \\ & I_{22}(\mu_1, \mu_2) \end{pmatrix} = \begin{pmatrix} E\left(-\frac{\partial^2 l}{\partial \mu_1^2}\right) & E\left(-\frac{\partial l}{\partial \mu_1}\frac{\partial l}{\partial \mu_2}\right) \\ & E\left(-\frac{\partial^2 l}{\partial \mu_2^2}\right) \end{pmatrix}. \tag{7}$$

The problem of interest is to know whether the normal approximation (6) is accurate. A good indicator of the accuracy of such approximation is given by Sprott (1973, 1980) which pointed out that if the elements of the FIM for the parameters are constant the normal approximation is very accurate, even for small or moderate-sized samples, which are common in reliability studies.

The presence of independence between the MLEs, which is a consequence of the orthogonality between them (Cox and Reid, 1987), is another important requirement for inferences about the parameters. The practical advantage of the orthogonality is that the MLE of a parameter does not depend on the other one. It may also lead to simplifications in the numerical determination of the MLEs.

The parameters μ_1 and μ_2 are orthogonal if, in (7), $I_{12}(\mu_1, \mu_2) = 0$. Then, for constructing of orthogonal parameters we solve differential equation given by (Cox and Reid,1987),

$$-I_{ij} = I_{ii}\frac{\partial \mu_i}{\partial \mu_j}, \quad i, j \in \{1, 2\}. \tag{8}$$

From now on, we focus on finding a parametrization where the desired properties stated above are observed.

4. Some Canditate Parametrizations

Firstly, let us introduce the parameter of interesting, θ_u, in the likelihood function (4). From (2) and (5),

$$\theta_u = \exp\left(Z_u + \beta_0 + \beta_1 X_u\right).$$

We then we obtain the Parametrization 1,

$$(\theta_u, \beta_1). \tag{9}$$

The corresponding likelihood function, from (4), is given by

$$L(\theta_u, \beta_1) \propto \theta_u^{-r} \exp\left\{-\beta_1 (a_1 - rX_u) - \theta_u^{-1} \sum_{i=1}^{k} A_i e^{-(Z_i - Z_u) + \beta_1 (X_u - X_i)}\right\}. \tag{10}$$

From (9), the Parametrization 2 is the logarithm transformation given by

$$(\delta = \log \theta_u \ , \ \beta_1). \tag{11}$$

The corresponding likelihood function is given by

$$L(\delta, \beta_1) \propto e^{-\delta r} \exp\left\{-\beta_1 (a_1 - rX_u) - e^{-\delta} \sum_{i=1}^{k} A_i e^{-(Z_i - Z_u) + \beta_1 (X_u - X_i)}\right\}. \tag{12}$$

The logarithm transformation allows to the new parameter δ to be unbounded, intuitively leading to a possible more accurate normal approximation for its MLE.

Another possibility is, from (5) and (9), to consider as the Parametrization 3 the orthogonal parametrization given by

$$\left(\varphi = \theta_u e^{-\beta_1 \overline{X}_u} \ , \ \beta_1\right), \tag{13}$$

where the parameter φ is the solution of the differential (8) with $\mu_1 = \beta_1$ and $\mu_2 = \theta_u$, and $\overline{X}_u = \sum_{i=1}^{k}(X_u - X_i)/\sum_{i=1}^{k} r_i$.

The corresponding likelihood function is given by

$$L(\varphi, \beta_1) \propto \varphi^{-r} \exp\left\{-\varphi^{-1} \sum_{i=1}^{k} A_i e^{-(Z_i - Z_u) + \beta_1 \left(X_u - X_i - \overline{\overline{X}}\right)}\right\}. \tag{14}$$

The problem with the parametrization (9) is that it do not lead to neither normality nor independence of the MLEs. And the parametrizations (11) and (13) lead to normality and to independence of the MLEs, respectively, but not to the two properties simultaneously. The second column of the Table 1 shows the FIM on the above parametrizations according to the likelihoods (11), (13), (15) and (17). The third and fourth columns show the effect of the parametrization on the orthogonality between the parameters and on the accuracy of the normal limiting distribution of the MLEs.

In order to have both properties at the same parametrization we propose, from (13), the following double parametrization

$$(\psi = \log \varphi \ , \ \beta_1), \tag{15}$$

as the Parametrization 4. We refer to (15) as a double parametrization since we consider a logarithm transformation over a orthogonal one.

The corresponding likelihood function is given by

$$L(\psi, \beta_1) \propto e^{-\psi r} \exp\left\{-e^{-\psi} \sum_{i=1}^{k} A_i e^{-(Z_i - Z_u) + \beta_1 \left(X_u - X_i - \overline{\overline{X}}\right)}\right\}. \tag{16}$$

Table 1. FIM for all parametrizations. In the second column $r = \sum_{i=1}^{k} r_i$, $b_1 = \sum_{i=1}^{k} r_i (X_i - X_u)$, $b_2 = \sum_{i=1}^{k} r_i (X_i - X_u)^2$, $c_1 = \sum_{i=1}^{k} r_i \left(X_u - X_i - \overline{X}\right)^2$ and $\overline{X} = \sum_{i=1}^{k} r_i (X_u - X_i)/r$.

Parametrization	FIM	Orthogonality	Normality
(θ_u, β)	$\begin{pmatrix} r/\theta_u^2 & b_1/\theta_u \\ b_1/\theta_u & b_2 \end{pmatrix}$	No	No
(δ, β)	$\begin{pmatrix} r & b_1 \\ b_1 & b_2 \end{pmatrix}$	No	Yes
(φ, β)	$\begin{pmatrix} r/\varphi^2 & 0 \\ 0 & c_1 \end{pmatrix}$	Yes	No
(ψ, β)	$\begin{pmatrix} r & 0 \\ 0 & c_1 \end{pmatrix}$	Yes	Yes

The last row of the Table 1 provides the FIM for the Parametrization 4, from where we observe both properties simultaneously, independence and normality of the MLEs.

From Table 1 we also observed that for all parametrizations the asymptotic variances are proportional to $r^{-1} = 1/\sum_{i=1}^{k} r_i$.

5. Simulation Study

In order to examine the coverage probabilities (CP) of the asymptotic confidence intervals obtained on different parametrizations we performed a simulation study. The study was based on samples generated from an exponential distribution with three different sets of parameters for the SRR (5), leading to the study of the most common SRRs considered in reliability, the Arrhenius, the Eyring and the power models, which were described at the end of Section 2. For the Eyring and Arrhenius model cases the parameters were fixed at $\beta_0 = -10$ and $\beta_1 = 7\text{x}10^4$, while they were fixed at $\beta_0 = 5.74$ and $\beta_1 = 0.6$ for the power model case. Two stress levels were considered. The parameters were subjectively fixed at the values above. However, a small sensitivity analysis was made by choosing others sets of parameters, but their choice do not modify substantially the results presented below and the correspondent results are omitted here.

We consider a particular situation where $n_i = n$ and $r_i = r_0$ for the two stress levels, where n and r_0 are a fixed value. While this is not the most common situation in practice, it can arise by design and is a natural base for theoretical comparisons. The sample sizes of each stress level were fixed at $n = 5, 10, 20, 30, 50, 70, 100$. A case studied was defined by the number of units at each stress level, by the percentage of censoring that was fixed at 0% (complete sample), 30%, 50% and 75% of censoring per stress level and by the model (Eyring, Arrhenius or Power) considered. Thus, eighty four different cases were simulated, each with $1,000$ samples.

For each sample we calculated the MLE of the usual stress level mean lifetime, θ_u and the 90% confidence intervals for this parameter based on the asymptotic normal theory and recorded whether the interval contain the true parameter values, whether the interval is located above or bellow of the parameter and the length of the interval.

The results of the study are summarized in plots of CPs versus sample size for confidence intervals and separately for lower and upper confidence bounds. Figures 1, 2 and 3 show the results based on the Eyring, Arrhenius and power models, respectively.

The results are consistently similar for all models. The CPs of the 90% confidence intervals for the parameters (top plots) are close to 0.90 when Parametrizations 2 and 4 are considered, even when the sample size is small and the censoring is heavy. The CPs however decrease more than 10% points when the number of units is small, the censoring is heavy and the Parametrizations 1 and 3 are considered. Clearly, the departure from nominal CP is worst when the confidence interval are based on the first paramerization, particularly for the censoring case. The separate CPs for the lower and upper confidence bounds (bottom plots) approach 0.95 when n increases only for Parametrizations 2 and 4. The under coverage of the confidence intervals are related to the under coverage of the upper bounds. We do not show the results related to the lengths of the confidence intervals based on all parametrizations, which reduce drastically when the number of units increases, proportional to n^{-1} when the samples are complete and to r_o^{-1} for censored samples.

We do not show the results related to the CPs of the 90% confidence intervals for the parameter β_1, which are very close to 0.90 even when the sample size is small and the censoring is heavy, regardless the considered parametrization.

6. Pressurized Containers Data

As a numerical example consider a benchmark data extracted from Barlow, Toland e Freeman (1986) on an ALT at pressurized containers of Kevlan/Epoxy 49. We have considered the test performed at two levels of pressure, V, equals to 3700 and 4000 *psi*, with 24 items at each level. Six censored values are observed at $V = 3700$. For purpose of illustration assume that interest focuses on the mean lifetime θ_u when $V_u = 3600$ *psi*.

An exponential distribution (1) for the lifetimes with the Eyring SRR (5) was suggested by goodness-of-fit procedures (Louzada-Neto, 1997). Figure 4 shows the 50%, 70%, 90% and 95% likelihood contours based on the four different parametrizations. Normality and independence of the MLEs are achieved only on Parametrization 4, (ψ, β), where we observe elliptical and symmetrical contours (right lower panel of Figure 4). Under the parametrization (ψ, β) the 90% confidence interval for θ_ux10^{-4} is $(0.70, 4.07)$ when the asymptotic normality of the MLEs is considered. The confidence limits were obtained by directly inverting (13) with β_1 substituted by its MLE. For having empirical evidence that the standard theory can be trusted we obtained the 90% percentile confidence interval for θ_ux10^{-4}, based on the double parametrization (ψ, β), equals to $(0.70, 4.06)$, which was obtained by considering a nonparametric bootstrap scheme where samples were obtained by direct resampling the original dataset (Davison and Hinkley, 1997). The bootstrap percentile confidence limits where obtained by taking the 0.025th and 0.975th quantiles of the empirical distribution of MLE. As a simple comparison the 90% confidence interval for θ_ux10^{-4} obtained by considering the parametrization (θ_u, β_1) is $(0.59, 2.79)$, which is very different of the interval obtained above via the parametrization (ψ, β).

7. Final Remarks

In this paper we study different parametrizations in order to obtain normality and orthogonality for the MLEs when only small or moderate samples are considered. We propose the double parametrization (ψ, β) which have such desired properties providing accuracy of the inferences for the model parameters of ALTs with a general SRR, bringing the double reparametrization approach to the attention of reliability analysis practitioners.

It is important to note that the accuracy of the asymptotical normality of the MLEs for small or moderate-sized samples is driven by parametrizations in which the parameters are unbounded. Besides, based on special reliability model cases we observed that the sample size and censoring affects the variances of the MLEs, which are proportional to n^{-1} when

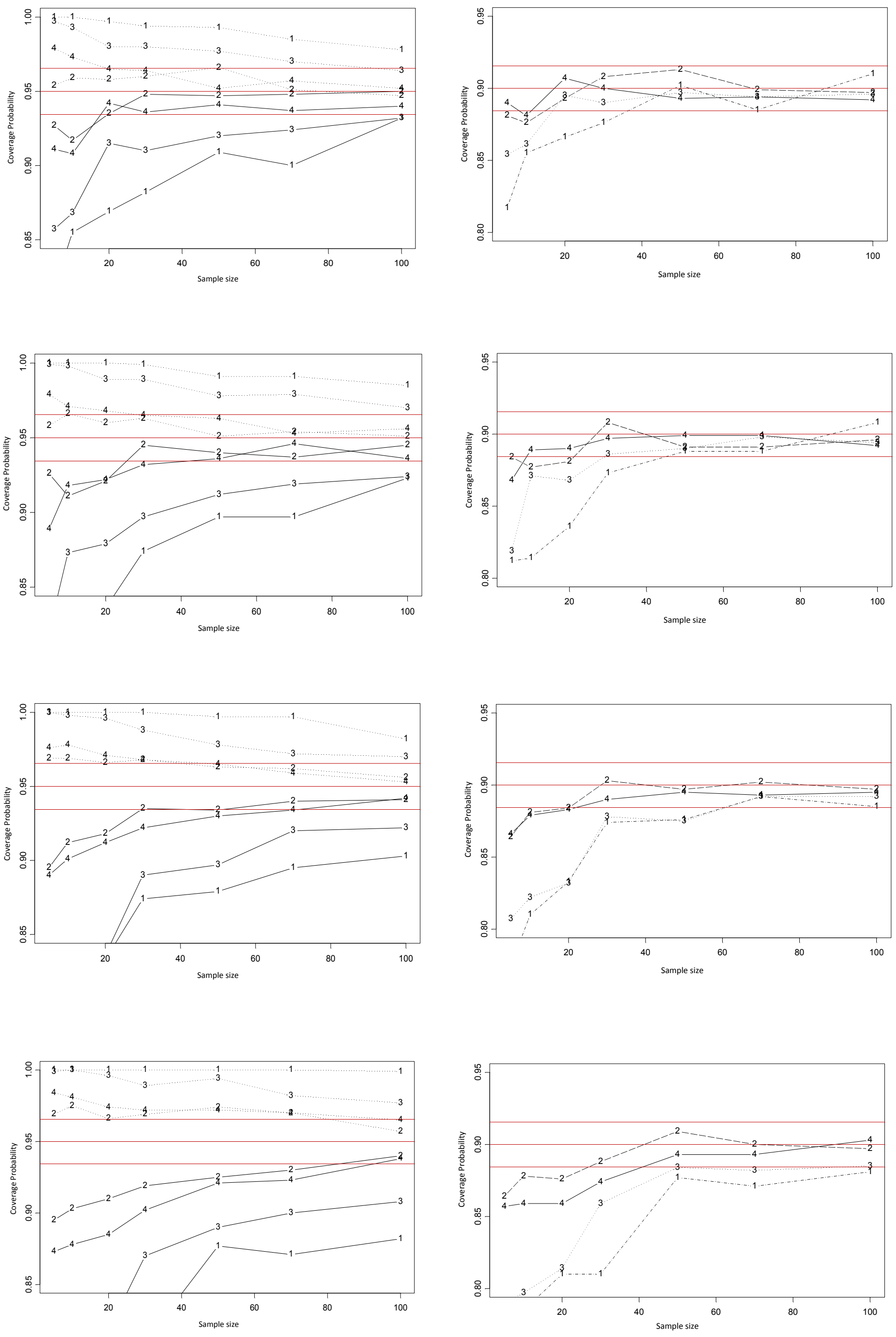

Figure 1. Results for the Eyring model. CPs of the 90% confidence interval for the parameters and of the lower and upper 90% confidence bounds versus n (sample size). Left plots are for confidence intervals (two-sided intervals) and right plots for lower (...) and upper (—) confidence bounds. The CPs for the confidence intervals based on Parametrizations 1, 2, 3 and 4 are indicated by the numbers 1, 2, 3 and 4, respectively. There were 1000 random samples in each of the 28 studies. In the top plots there is a horizontal line at CP= 0.90 and two horizontal lines at CP= 0.884 and 0.916 which correspond, respectively, to the lower and upper bounds of the 99% confidence interval of the CP= 0.90. If a confidence interval has exact coverage of 0.90, roughly 99% of the observed coverages should be between these lines. In the bottom plots there is a horizontal line at CP= 0.95 and two horizontal lines at CP= 0.934 and 0.966 which correspond, respectively, to the lower and upper bounds of the 99% confidence interval of the CP= 0.95. If a lower or upper confidence bound has exact coverage of 0.95, roughly 99% of the observed coverages should be between these lines.

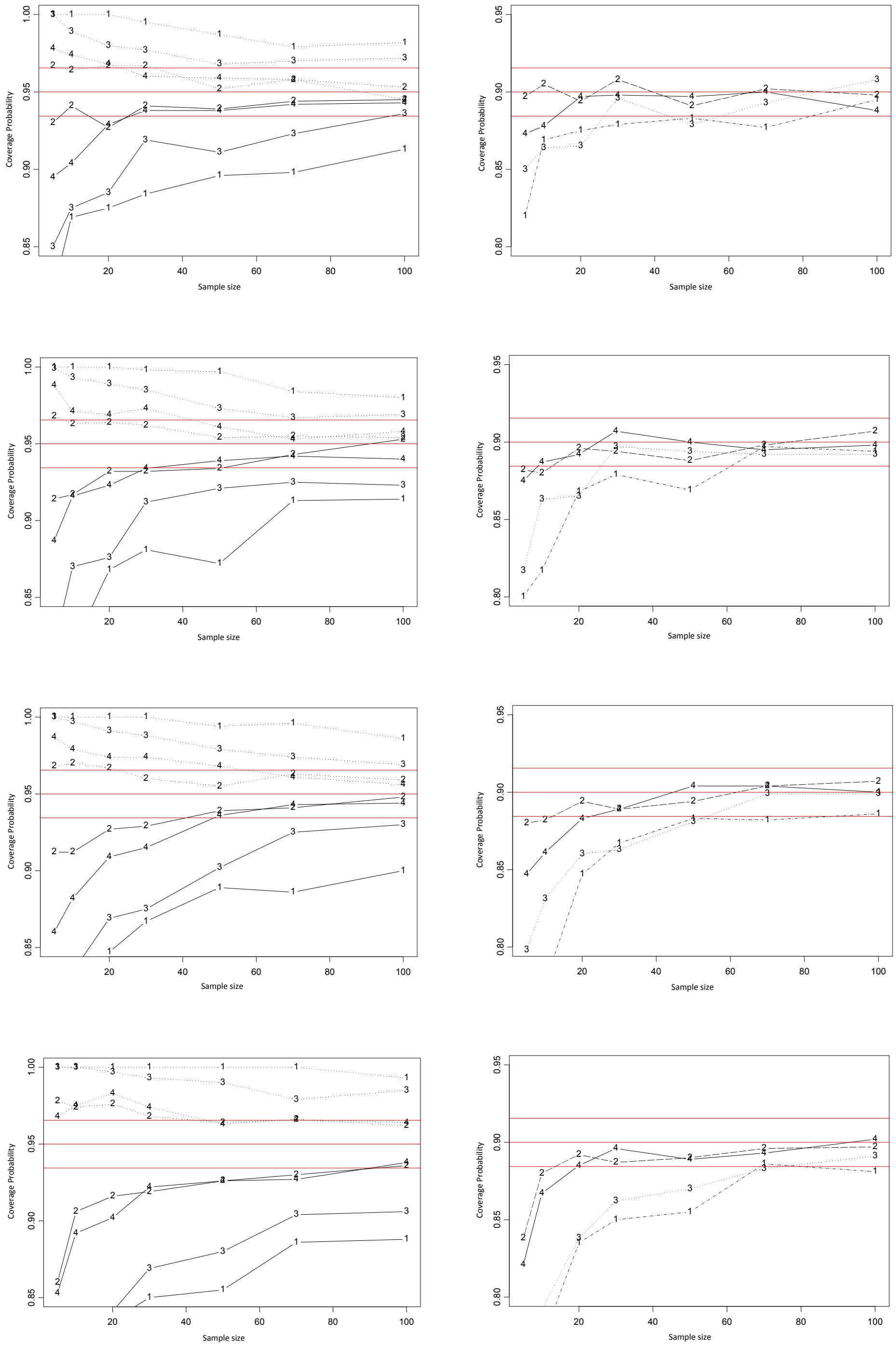

Figure 2. Results for the Arrhenius model. CPs of the 90% confidence interval for the parameters and of the lower and upper 90% confidence bounds versus n (sample size). Left plots are for confidence intervals (two-sided intervals) and right plots for lower (...) and upper (—) confidence bounds. The CPs for the confidence intervals based on Parametrizations 1, 2, 3 and 4 are indicated by the numbers 1, 2, 3 and 4, respectively. There were 1000 random samples in each of the 28 studies. In the top plots there is a horizontal line at CP= 0.90 and two horizontal lines at CP= 0.884 and 0.916 which correspond, respectively, to the lower and upper bounds of the 99% confidence interval of the CP= 0.90. If a confidence interval has exact coverage of 0.90, roughly 99% of the observed coverages should be between these lines. In the bottom plots there is a horizontal line at CP= 0.95 and two horizontal lines at CP= 0.934 and 0.966 which correspond, respectively, to the lower and upper bounds of the 99% confidence interval of the CP= 0.95. If a lower or upper confidence bound has exact coverage of 0.95, roughly 99% of the observed coverages should be between these lines.

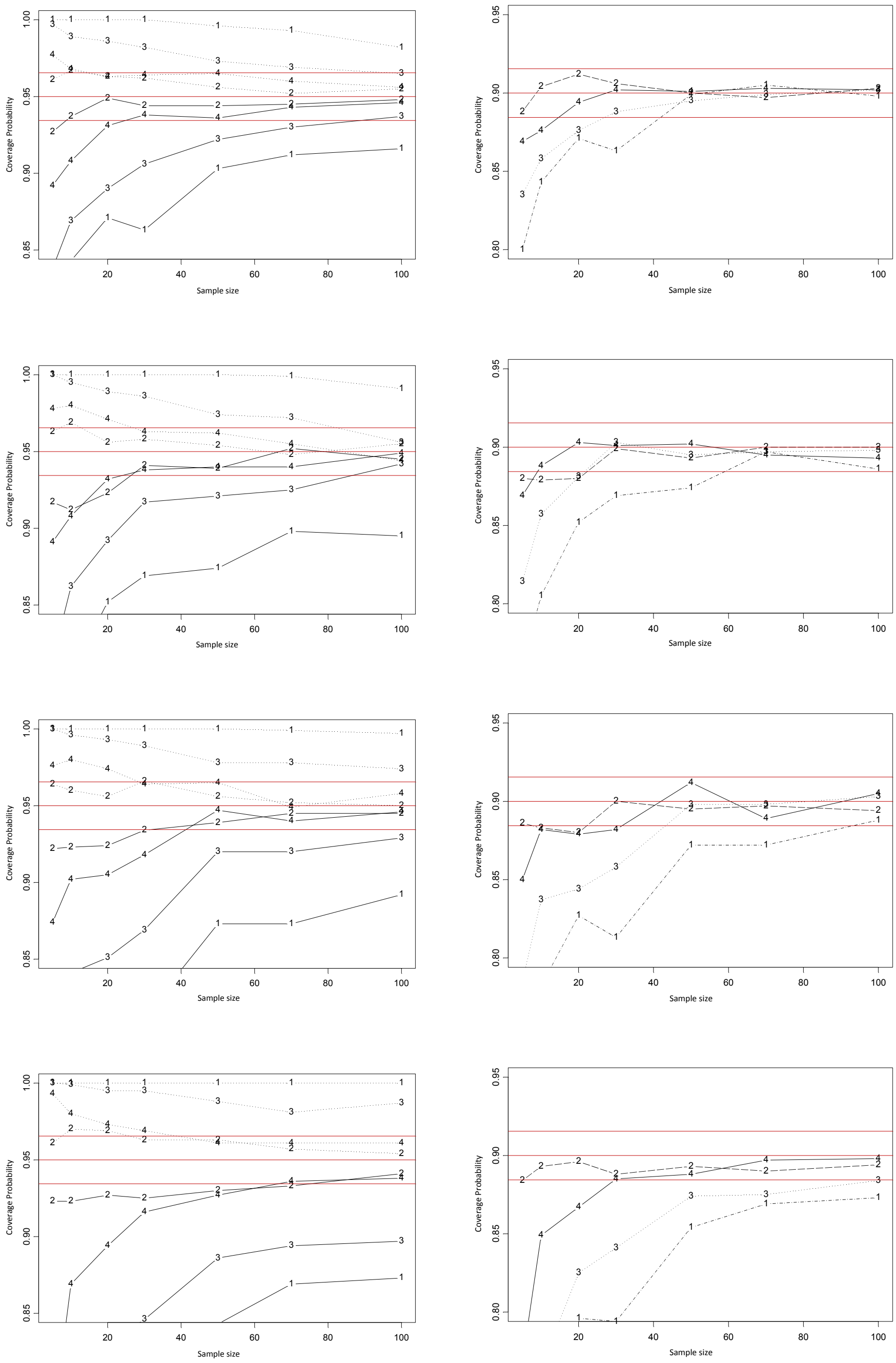

Figure 3. Results for the power model. CPs of the 90% confidence interval for the parameters and of the lower and upper 90% confidence bounds versus n (sample size). Left plots are for confidence intervals (two-sided intervals) and right plots for lower (...) and upper (—) confidence bounds. The CPs for the confidence intervals based on Parametrizations 1, 2, 3 and 4 are indicated by the numbers 1, 2, 3 and 4, respectively. There were 1000 random samples in each of the 28 studies. In the top plots there is a horizontal line at CP= 0.90 and two horizontal lines at CP= 0.884 and 0.916 which correspond, respectively, to the lower and upper bounds of the 99% confidence interval of the CP= 0.90. If a confidence interval has exact coverage of 0.90, roughly 99% of the observed coverages should be between these lines. In the bottom plots there is a horizontal line at CP= 0.95 and two horizontal lines at CP= 0.934 and 0.966 which correspond, respectively, to the lower and upper bounds of the 99% confidence interval of the CP= 0.95. If a lower or upper confidence bound has exact coverage of 0.95, roughly 99% of the observed coverages should be between these lines.

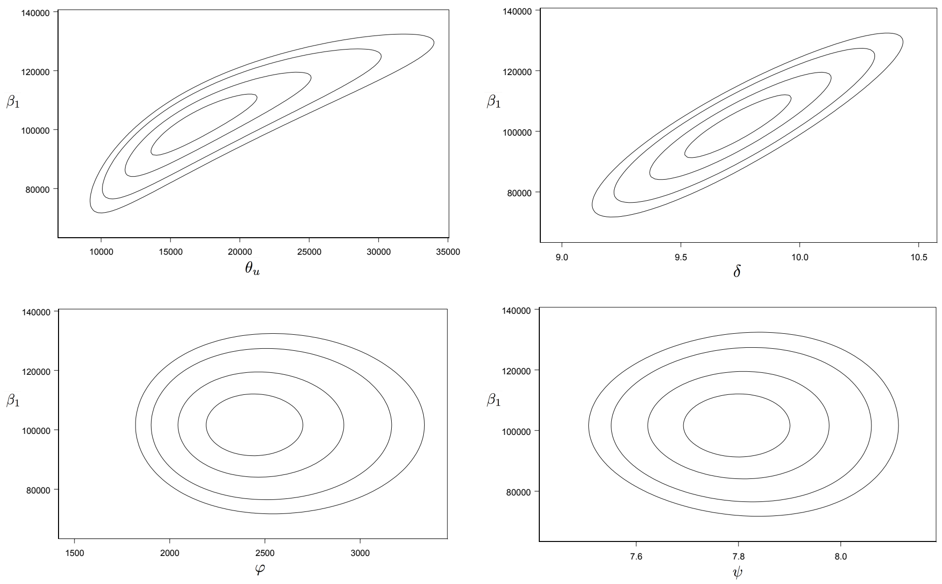

Figure 4. ALT at pressurized vasels of Kevlan/Epoxy 49. 50%, 70%, 90% and 95% likelihood contours based on the four different parametrizations. Upper left plot is for the Parametrization 1, upper right plot is for Parametrization 2, lowe left plot is for the Parametrization 3 and lower right plot is for Parametrization 4.

the sample is complete and to r_o^{-1} when presence of censoring is observed. Finally, our simulation study revels that the CPs of the confidence intervals obtained by considering the double parametrization (ψ, β) are close to the nominal CP even when the sample size is small and the censoring is heavy, and are therefore preferred.

Although we assumed an exponential distribution as the lifetime model, more general lifetime distributions, such as the Weibull, log-normal, log-logistic, among others, could be considered in principle. However, the degree of difficulty in the calculations may increase considerably.

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