<u>Statistics Education</u>
Teaching Paper

Training school teachers to teach probability: reflections and challenges

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Abstract

Although probability is today part of the mathematics curricula for primary and secondary schools in many countries, the specific training to teach probability is far from being an universal component of pre-service courses for mathematics teachers responsible of this training. In this paper, we analyse the reasons why the teaching of probability is difficult for mathematics teachers. In addition, we describe the contents needed in the preparation of teachers to teach probability and suggest possible activities to carry out this training.

1. Introduction

The reasons for including probability in schools have been repeatedly highlighted over the past years; see, e.g., Gal (2005), Franklin et al. (2005) and Jones (2005). These reasons are related to the usefulness of probability for daily life, its instrumental role in other disciplines, the need for a basic stochastic knowledge in many professions, and the important role of probability reasoning in decision making. Students will meet randomness not only in the mathematics classroom, but also in biological, economic, meteorological, political and social activities (games and sports) settings. All these reasons explain why probability has recently been included in the primary school curriculum in many countries since very early ages and why the study of probability continues later through secondary and high school and universities studies.

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Changes in what is expected in the teaching of probability and statistics do not just concern the age of learning or the amount of material, but also the approach to teaching. Until recently, the school stochastic (statistics and probability) curriculum was reduced to a formula-based approach that resulted in students who were ill prepared for tertiary level statistics and adults who were statistically illiterate. The current tendency even for primary school levels is towards a data-orientated teaching of probability, where students are expected to perform experiments or simulations, formulate questions or predictions, collect and analyse data from these experiments, propose and justify conclusions and predictions that are based on data; see, e.g., NCTM (2000), Parzysz (2003) and MEC (2006a,b). As argued in Batanero et al. (2005), these changes force us to reflect on the teaching of chance and probability.

The importance of developing stochastic thinking and not just stochastic knowledge in the students is being emphasized in many curricula. Indeed, some authors argue that stochastic reasoning is different from mathematical reasoning, both of them being essential to modern society and complementing each other in ways that strengthen the overall mathematics curriculum for students; see Scheaffer (2006).

Changing the teaching of probability in schools will depend on the extent to which we can convince teachers that this is one of the most useful themes for their students, as well as on the correct preparation of these teachers. Unfortunately, several authors agree that many of the current programmes do not yet train teachers adequately for their task to teach statistics and probability; see, e.g., Begg and Edwards (1999), Franklin and Mewborn (2006), Borim and Coutinho (2008) and Chick and Pierce (2008). Even when many prospective secondary teachers have a major in mathematics, they usually study only theoretical (mathematical) statistics and probability in their training. Few mathematicians receive specific training in applied statistics, designing probability investigations or simulations, or analysing data from these investigations. These teachers also need some training in the pedagogical knowledge related to the teaching of probability, where general principles that are valid for geometry, algebra or other areas of mathematics cannot always be applied. The situation is even more challenging for primary teachers, few of whom have had suitable training in either theoretical or applied probability, and traditional introductory statistics courses will not provide them with the didactical knowledge they need; see Franklin and Mewborn (2006).

Research in statistics education shows that textbooks and curriculum documents prepared for primary and secondary teachers might not offer enough support. Sometimes they present too narrow a view of concepts (for example, only the classical approach to probability is shown). In addition, applications are at other times restricted to games of chance. Finally, in some of them the definitions of concepts are incorrect or incomplete; see Cañizares et al. (2002). There are also exceptional examples and experiences of courses specifically directed to train teachers in different countries some of them based on theoretical models of how this training should be carried out; see, e.g., Kvatinsky and Even (2002), Batanero et al. (2004) and Garfield and Everson (2009). Although evaluation of the success of such courses is in general based on small samples or subjective data, they do provide examples and ideas for teacher educators.

The aim of this paper is to reflect on some specific issues and challenges regarding the education of teachers to teach probability at school level. In Section 2, we analyse the specific features of probability and the different meanings of this concept that should be taken into account when teaching probability at school level. In Section 3, we summarise the scarce research related to teachers' beliefs and knowledge as regards probability, which are not always adequate. In Section 4, we then discuss possible activities that can contribute to the education of teachers to teach probability. Finally, in Section 5, we conclude this work with some personal recommendations.

2.   The Nature of Probability

A main theme in preparing teachers is discussing with them some epistemological problems linked to the emergence of probability, because this reflection can help teachers to understand the students' conceptual difficulties in problem solving. Probability is a young area and its formal development was linked to a large number of paradoxes, which show the disparity between intuition and mathematical formalization in this field; see Borovcnik and Peard (1996). Counterintuitive results in probability are found even at very elementary levels (for example, the fact that having obtained a run of four consecutive heads when tossing a coin does not affect the probability that the following coin will result in heads is counterintuitive). Difficulty at higher levels is also indicated by the fact that even though the Kolmogorov axioms were generally accepted in 1933, professional statisticians still debate about different views of probability and different methodologies of inference.

Another difference is reversibility. In arithmetic or geometry, an elementary operation (like addition) can be reversed and this reversibility can be represented with concrete materials. This is very important for young children, who still are very linked to concrete situations in their mathematical thinking. For example, when joining a group of two apples with another group of three apples, a child always obtain the same result (five apples). However, if separating the second set from the total, he/she always returns to the original set, no matter how many times this operation is repeated. These experiences are very important to help children progressively abstract the mathematical structure behind them. In the case of random experiment, we obtain different results each time the experiment is carried out and the experiment cannot be reversed (we cannot get the first result again when repeating the experiment). This makes the learning of probability comparatively harder for children.

Of particular relevance for teaching probability are the informal ideas that children and adolescents assign to chance and probability before instruction and that can affect subsequent their learning. As an example, Truran (1995) found substantial evidence that young children do not see random generators such as dice or marbles in urns as having constant properties and consider a random generator has a mind of its own or may be controlled by outside forces. There is also evidence that people at different ages maintain probability misconceptions that are hard to eradicate with only a formal teaching of the topic; see Jones et al. (2007). Even though simulation or experimentation with random generators, such as dice and coins have a very important function in stabilizing children's intuition and in materializing probabilistic problems, these experiences do not provide the key to how and why the problems are solved. It is only with the help of combinatorial schemes or tools like tree diagrams that children start to understand the solution of probabilistic problems; see Fischbein (1975). This indicates the complementary nature of classical and frequentist approaches to probability.

Another reason for difficulty in the field of probability is that the meaning of some concepts is sometimes too tied to applications. For instance, although independence is mathematically reduced to the multiplicative rule, this definition does not include all the causality problems that subjects often relate to independence nor always serve to decide if there is independence in a particular experiment.

2.1   Meanings of probability

Different meanings also are linked to the concept of probability, which depends on the applications of this concept in real situations and that are relevant in the teaching of the topic, such as:

- INTUITIVE MEANING. Ideas related to chance and probability and games of chance emerged very early in history in many different cultures, as they appear in young children. These conceptions were too imprecise, so that we need the fundamental idea of assigning numbers to uncertain events to be able to compare their likelihood and thus being able to apply mathematics to the wide world of uncertainty. Hacking (1975) indicated that probability had a dual character since its emergence. A statistical side was concerned with the stochastic rules of random processes, while the epistemic side views probability as a degree of belief. This duality was present by many of the authors who helped to develop the theory of probability. For example, while Pascal's solution to games of chance reflects an objective probability, his argument to support belief in the existence of God presents probability as a personal degree of belief.

- CLASSICAL MEANING. The first probability problems were linked to games of chance, where equiprobability is natural in many cases. For this reason, Laplace suggested reducing all the events in a random experiment to a certain number of equally probable possible cases and considered that probability is simply a fraction whose numerator is the number of favourable cases and whose denominator is the number of all cases possible; see Laplace (1985, p. 28). This definition was criticised because although equiprobability is clear when playing a chance game, it can hardly be found apart from games of chance.

- FREQUENTIST MEANING. The convergence of frequencies for an event, after a large number of identical trials of random experiments, had been observed over several centuries. Bernoulli's proof that the stabilized value approaches the classical probability, was interpreted as a confirmation that probability was an objective feature of random events. Given that stabilized frequencies were observable, they can be considered as approximated physical measures of this probability. However, the frequentist approach does not provide the exact value of the probability for an event and we cannot find an estimate of the same when it is impossible to repeat an experience a very large number of times. It is also difficult to decide how many trials are needed to get a good estimation for the probability of an event; see Batanero et al. (2005).

- SUBJECTIVE MEANING. Bayes' formula permitted the finding of the probabilities of various causes when one of their consequences is observed. The probability of such a cause would thus be prone to revision as a function of new information and would lose its objective character postulated by the above conceptions. Keynes, Ramsey and de Finetti described probabilities as personal degrees of belief, based on personal judgment and information about experiences related to a given outcome. They suggested that the possibility of an event is always related to a certain system of knowledge and is thus not necessarily the same for all people. The fact that repeated trials were no longer needed was used to expand the field of applications of probability theory. However, the controversy about the scientific status of results that depends on personal judgments still remains.

- MATHEMATICAL MEANING. Throughout the 20th century, different mathematicians contributed to the development of the mathematical theory of probability. Borel's view of probability as a special type of measure was used by Kolmogorov, who applied sets and measure theories to derive a satisfactory set of axioms, which was accepted by different schools regardless of their philosophical interpretation of the nature of probability. Probability theory proved its efficiency in many different fields, but the particular models used are still subjected to heuristic and theoretical hypotheses, which need to be evaluated empirically.

When teaching probability, these different meanings of probability should be progressively taken into account, starting by the students' intuitive ideas of chance and probability and subjective view of probability as a degree of belief. In order to connect statistics and probability, some curricula (e.g., in France) suggest implementing an experimental approach to probability, through experiments and simulations with the purpose of preparing students to understand the law of large numbers and to grasp the connections between the notions of relative frequency and probability. In simple chance games where equiprobability is a reasonable assumption, the classical approach will be useful. Since understanding is a continuous constructive process, students should progressively acquire and relate the different meanings of the concept and at the final stage reach to the mathematical formalism and axiomatic.

In summary, probability is difficult to teach, because we should not only present different probabilistic concepts and show their applications, but we have to go deeper into wider questions, consisting of how to interpret the meaning of probability, how to help students develop correct intuitions in this field and deal with controversial ideas, such as causality.

3. Training Teachers to Teach Probability

Although teachers do not need high levels of mathematical knowledge, they do require a profound understanding of the basic mathematics we teach at school level, including a deep grasp of the interconnections and relationships among different aspects of this knowledge (for example, understanding the different meanings associated with probability); see Ma (1999). In addition, teachers need good attitudes towards the correct beliefs about a topic, and a good professional knowledge for teaching. Next, we analyse each of these components for the case of probability.

3.1 Teachers' attitudes and beliefs

Teachers' attitudes and beliefs about how a topic should be taught are key factors to assure success of curricular reforms, since that these may be transmitted to the students. Steinbring (1990) suggested that it is important for mathematics teachers to understand the particular nature of the stochastic knowledge. While traditional mathematics teaching is based on a hierarchical and cumulative amount of concepts, which are learnt in a linear sequence, stochastic knowledge is more complex so that systemic and stochastic problems must include much more interpretative activities than other areas of mathematics. For instance, to understand what a random variable is, a person needs to assume a model for randomness, a concept linked to many philosophical interpretations. Moreover, teachers also have personal views of what are important instructional contents and goals, so that the teaching that a student receives for a similar level and curriculum might differ considerably depending on whether a teacher has a static view versus a dynamic view of mathematics, or whether the teacher prefers to teach formal mathematics versus mathematical applications; see Eichler (2008). In summary, it is important to promote in the teacher the view that the stochastic knowledge has its own specificity and should be linked to applications in order to develop the teacher's self-competence to teach probability and statistics. At the same time, we have to produce conditions for integrating the stochastic knowledge in the school, while widening the teachers' views about mathematics.

3.2 Teachers' probabilistic knowledge

Some of the activities that teachers regularly engage involve mathematical reasoning and thinking. Such activities consider figuring out what students know, choosing and managing representations of mathematical ideas, selecting and modifying textbooks and deciding among alternative courses of action; see Ball et al. (2001, p. 243). Consequently, teachers' instructional decisions in the teaching of probability are dependent on the teacher's probabilistic knowledge. This is cause for concern when paired with evidence that mathematics teachers, especially at the primary school level, tend to have a weak understanding of probability. For example, the study by Begg and Edwards (1999) found that only about two-thirds of in-service and pre-service primary school teachers understood equally probable events and very few understood the concept of independence. Batanero et al. (2005) analysed results from an initial assessment based on sample of 132 pre-service teachers in Spain that showed they frequently have three probabilistic misconceptions: representativeness, equiprobability and the outcome approach. In a research conducted by Borim and Coutinho (2008), the following results were obtained. First, secondary school teachers' predominant reasoning about variation in a random variable was verbal, which did not allow these teachers to teach their students the meaning of measures such as standard deviation, restricting them to the teaching of algorithms. Second, none of the teachers integrated process reasoning, which would relate the understanding of mean, deviations from the mean, the interval of k standard deviations from the mean and the density estimation of frequency in that interval. Canada (2008) examined how pre-service teachers reasoned about distributions as they compared graphs of two data sets and found that almost 35% of the sample found no real difference when average was similar but spread was quite different.

Few teachers have prior experience with conducting probability experiments or simulations and may have difficulty implementing an experimental approach to teaching probability; see Stohl (2005). In a research conducted by Lee and Hollebrands (2008), although the participant teachers engaged students in investigations based on probability experiments, they often missed opportunities for deepening students' reasoning. Teachers' approaches to using empirical estimates of probability did not foster a frequentist conception of probability as a limit of a stabilized relative frequency after many trials. Teachers almost exclusively chose small samples sizes and rarely pooled class data or used representations supportive of examining distributions and variability across collections of samples so they failed to address the heart of the issue.

3.3 Teachers' professional knowledge

Wide probability knowledge, even when essential, is not enough for teachers to be able to teach probability. As argued by Ponte and Chapman (2006), we should view teachers as professionals, and ground teacher education in professional practice, making all elements of practice (preparing lessons, tasks and materials, carrying out lessons, observing and reflecting on lessons) a central element in the teacher education process. In fact, research focused on teacher's training is producing a great deal of information about this "professional knowledge", which includes the following complementary aspects:

- Epistemological reflection on the meaning of concepts to be taught (e.g., reflection on the different meaning of probability). For the particular case of the stochastic knowledge, Biehler (1990) also suggested that teachers need a historical, philosophical, cultural and epistemological perspective on this knowledge and its relationships to other domains of science.

- Experience in adapting this knowledge to different teaching levels and students' various levels of understanding. This includes, according to Steinbring (1990), organizing and implementing teaching, experiencing students' multiple forms of work and understanding experiments, simulations and graphical representations not just as methodological teaching aids, but rather as essential means of knowing and understanding.

- Critical capacity to analyse textbooks and curricular documents.

- Prediction of students' learning difficulties, errors, obstacles and strategies in problem solving (e.g., students strategies in comparing two probabilities and students' confusion between the two terms in a conditional probability).

- Capacity to develop and analyse assessment tests and instruments and interpret students' responses to the same.

- Experience with good examples of teaching situations, didactic tools and materials (e.g., challenging and interesting problems, Galton board, simulation, calculators, etc.).

Some significant issues related to the professional knowledge of teachers are whether teachers are able or not to (i) recognize what concepts can be addressed through a particular resource or task, and (ii) implement effective learning in the classroom with them. However, the current preparation of teachers do not always assure this professional knowledge. For example, research conducted by Chick and Pierce (2008) found the teachers' lack of professional knowledge was evident in their approaches to the lesson-planning task, failing to bring significant concepts to the fore, despite all the opportunities that were inherent in the teaching tasks and resources.

4.   POSSIBLE ACTIVITIES TO TRAIN TEACHERS IN PROBABILITY

It is important to find suitable and effective ways to teach this mathematical and professional knowledge to teachers. Since students build their knowledge in an active way, by solving problems and interacting with their classmates, we should use this same approach in training the teachers especially if we want that they use a constructivist and social approach in their teaching; see Jaworski (2001). An important view is that we should give teachers more responsibility in their own training and help them to develop creative and critical thinking. That is why we should create suitable conditions for teachers to reflect on their previous beliefs about teaching and discuss these ideas with other colleagues.

One fundamental learning experience that teachers should have to develop their probability thinking is working with experiments and investigations. To teach inquiry, teachers need skills often absent in mathematics classrooms: such as ability to cope with ambiguity and uncertainty; re-balance between teacher guidance and student independence and deep understanding of disciplinary content. Some other approaches in the training of teachers include:

- TEACHERS' COLLECTIVE ANALYSIS AND DISCUSSION OF THE STUDENTS' RESPONSES, BEHAVIOR, STRATEGIES, DIFFICULTIES AND MISCONCEPTIONS WHEN SOLVING PROBABILITY PROBLEMS. Groth (2008) suggested that teachers of stochastics must deal with two layers of uncertainty in their daily work. The first layer relates to disciplinary knowledge. Uncertainty is also ubiquitous in teaching because of the unique and dynamic interactions among teacher, students, and subject matter in any given classroom. Hence, teachers must understand and navigate the uncertainty inherent to both stochastics and the classroom simultaneously in order to function effectively. Case discussion among a group of teachers where they offer and debate conjectures about general pedagogy, mathematical content, and content-specific pedagogy can help teachers challenge one another's claims and interpretations; see Groth (2008).

- PLANNING A LESSON TO TEACH STUDENTS SOME CONTENT USING A GIVEN INSTRUC-
  TIONAL DEVICE TO DEVELOP PROBABILITY AND PROFESSIONAL KNOWLEDGE OF
  TEACHERS; see Chick and Pierce (2008). Since teachers are asked to teach probability
  for understanding, it is essential that they experience the same process as their students.
  One way to do this is to have the students play the role of the learner and the teacher
  at the same time, going through an "actual teacher as learner practice". If they had the
  chance to go through such a lesson as a learner and at the same time look at it from
  the point a view of a teacher, chances are that they will try it out in their own classrooms.

- PROJECT WORK. New curriculum and methodology guidelines suggest that, when
  teachers are involved in research projects, it can change how mathematics is experienced
  in the classroom, especially in connection to stochastics. Inquiry is a well accepted
  (but not always implemented) process in other school subjects, like science and social
  studies, but it is rarely used in a mathematics classroom (where statistics is usually
  taught). Moreover, when time available for teaching is scarce a formative cycle where
  teachers are first given a project to work with and then carry out a didactical analysis
  of the project can help to simultaneously increase the teachers' mathematical and
  pedagogical knowledge and at the same time provides the teacher educator with
  information regarding the future teachers' previous knowledge and learning; Godino et
  al. (2008).

- WORKING WITH TECHNOLOGY. We can also capitalize on the ability of some software
  to be used as a tool-builder to gain conceptual understanding of probability ideas. Lee
  and Hollebrands (2008) introduced a design that used technology both as amplifiers
  and reorganisers to engage teachers in tasks that simultaneously developed their un-
  derstanding of probability with technology and provided teachers with experience first
  hand about how technology tools can be useful in fostering stochastical thinking. In the
  experiences of Batanero et al. (2005), simulation helped to train teachers simultaneously
  in probability content and its pedagogy, since it helps to improve the teachers' prob-
  abilistic knowledge, while making them conscious of incorrect intuitions within their
  students and themselves.

5.   FURTHER REFLECTIONS

Teachers need support and adequate training to succeed in achieving an adequate equi-
librium of intuition and rigour when teaching probability. Unfortunately, due to time
pressure, teachers do not always receive a good preparation to teach probability in their
initial training. It is important to convince teacher educators that stochastics is an essen-
tial ingredient in the training of teachers. Moreover, despite the acknowledged fact that
probability is distinct and different from other areas of mathematics and the implied need
to provide mathematics teachers with a special preparation to teach this topic, it is pos-
sible to connect the stochastic and mathematical preparation of teachers when time for
training teachers is scarce.

Finally, much more research is still needed to clarify the essential components in the
preparation of teachers to teach probability, identify adequate methods, and establish ap-
propriate levels at which each component should be taught. The significant research efforts
focusing that have focused on mathematics teacher education and professional development
in the past decade (see Llinares and Krainer, 2006; Ponte and Chapman, 2006; Hill et al.,
2007; Wood, 2008) have not been reflected in statistics education. This is an important
research area that can contribute to improve statistics education at school level.

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